Volatility on the Crypto-currency Market: A Copula-GARCH Approach *

Kensley Blaise†

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Abstract

This research investigates whether crypto-currency returns correlate with expected uncertainty related to the global economy or other risky markets. In a first attempt, we use the copula framework to estimate the dependence magnitude between returns on the crypto-currency market, the interest rate spread, the breakeven inflation and the volatility index from the S&P500 options (VIX). Our results show that uncertainty information about future policy and the state of the economy contained in the interest rate spread bears no importance in crypto-currency price fluctuations. However, we find a pattern, although relatively small, for high estimated crypto-currency returns volatility to overlap with low VIX values. On another level, we find evidence that dynamic time series models can improve our understanding of price fluctuations on the crypto-currency market. We estimate a 5.6 percentage points increase of today's log-returns on the crypto-currency market for each one percentage point increase of yesterday's breakeven inflation. The effect is instantaneous and about 12 percentage points in recent time periods (2020-2022).

Keywords: Crypto-currency; government bonds; volatility modelling; copula

1 Introduction

The crypto-currency's secondary market rise is a singular case study in the financial literature. The market has gone from 0 in valuation in January 2010 to more than 2 trillion United States

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†School of Economics, University of East Anglia, NR4 7TJ Norwich (United Kingdom). Email address:k.blaise@uea.ac.uk

Dollar (USD) in March 2022. The unconventionally high returns on crypto-currencies is a possible reason for this expansion. From 2017 to 2021, the annual returns on Bitcoin (BTC) averaged 76%. In contrast, the average annual returns on the world's largest equity index, the S&P500, sat at around 15%. The exceedingly high returns coupled with low entry barriers have turned crypto-currencies into an attractive class of assets for retail investors. More recently, the crypto-currency market (CM) activity has also been amplified with an influx of institutional investments. As a consequence, this nascent market has been subjected to important scrutiny work from regulators and academics alike.

A major impediment with crypto-currencies is the unstable fluctuation around the mean returns. The average annual volatility of the BTC, proxied by the standard deviation of the returns distribution, oscillated around 77% between 2012 and 2021. We evaluated the average volatility of the S&P500 at 20% over the same time period. So, the BTC price is approximately four times more volatile than the S&P500 index. By traditional standards, the crypto-currency trading is an extremely high-risk financial activity.

Are there financial and economic drivers to explain price fluctuations on the CM? A strand of the emerging literature identifies interest rates on government bonds as a major candidate to explain crypto-currency prices (Karau 2021, Aboura 2022). The same argument is also prevalent in economically inclined newspapers (The Economist 2022, Financial Times 2022). In a nutshell, the reasoning supposes that higher interest rates on government bonds crowd capital out of the CM. Similarly, low interest rates on government securities increase both investors' risk-taking attitude and the attractiveness of the CM. The latter reasoning is similar to the risk-shifting mechanism studied in Rajan (2006) and Borio & Zhu (2012). Put differently, the explanation posits a trade-off between holding crypto-currencies and government bonds, which is a variation of the classical trade-off in portfolio construction with risky and risk-free assets.

Our research addresses two levels of inconsistencies in the current literature. On the one hand, it is likely the interest rate channel identified in the literature arises from an inadequate interpretation. For instance, Aboura (2022) argues that the March 2020 interest rate cut in the US was instrumental to the subsequent crypto-currency bullish run. However, the same period witnessed the inception of multiple fiscal transfer packages directed to households and small enterprises across the globe. Higher household savings, driven in part by the pandemic-related restrictions, may have instead dictated retail investors' preference for crypto-currencies [1]. On the other hand, a crypto-currency price response to interest rate change does not lead to clear-cut conclusions. Publications in this area often report incoherent crypto-currency price reactions, which depends on various policy set-ups and the country considered for the object of the analysis (Karau 2021, Aboura 2022). Consequently, these contrasts lessen the relevance of these studies in practical decision-making related to the CM.

Our study re-examines the dependence between price variation on the CM and interest rate

[1] See Dossche et al. (2021) for an overview on household savings increase in the euro area during the pandemic and the allocation of a sizable part of them to financial investments.

movements. Our focus is on the direction and the steepness of the yield curve, meaning the sign and the magnitude of the curve slope. The latter accounts for both intertemporal change in interest rates and expectations about market conditions, which is the basis of the expectations theory. Implicitly, we are testing for whether the direction of the yield curve is informative for returns fluctuation. On another level, we also look at the dependence between the CM and other risky markets. For the risky market extension, we use the Volatility Index (VIX) as a measure of expected volatility. Similar to the government bond slope reasoning, the VIX incorporates uncertainty information about 500 leading firms in the US economy (Bekaert et al. 2013). Our last bivariate dependence analysis involves the price variation on the CM and the US 5-year breakeven inflation. As a measure of expected inflation, the breakeven variable is important to gauge how investors align their decisions with future market conditions. The existence of fluctuations synchronization or asymmetry between price movements on the CM and the variables described in this section would be a starting point to rationalize investments decisions regarding the CM.

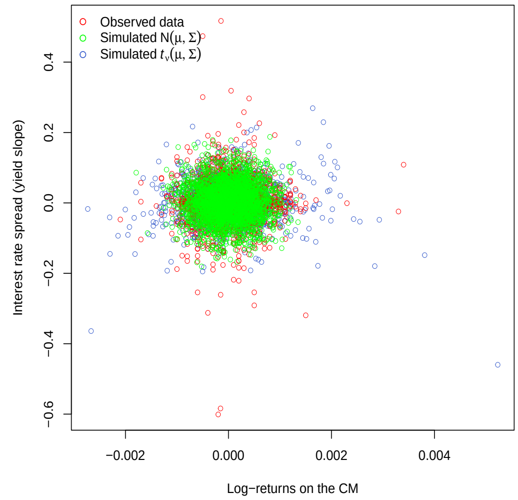
We use the copula framework to estimate the dependence between returns on the CM and the three other variables, meaning the yield curve slope or the interest rate spread, the VIX and the breakeven inflation. The copula is an important approach to study the dependence between continuous random variables when the pearson correlation and related techniques break down. Patton (2006) gives an illustration of the copula technique importance to analyze the case of an asymmetric relationship between exchange rates. Other econometric use of the copula method in finance and economics is described in Patton (2006). In this paper, our main objective is to measure the co-movement between price fluctuations on the CM and the variables mentioned above. The technique also allows the decomposition and visualization of the dependence between the variables in terms of extreme events, labelled as upper and lower tail dependence in the copula literature. For example, a tendency for high extreme returns on the CM to cluster with high extreme yield slope values would be a situation of an upper tail dependence, with the converse explanation for the lower tail dependence.

We illustrate the existence of extreme events between log-returns on the CM and the slope of the US yield curve in Figure 1 [2]. At first, the two observed series are plotted against each other. Then, we extract the mean and the correlation estimate of the observed variables to simulate bivariate normal and student's t distributions that would likely arise from such estimates. Figure 1 shows a clear departure from normality and the existence of extreme events that are best captured by the bivariate student's t. Using the copula technique, we provide estimates of the magnitude and the statistical significance of these extreme events.

The dataset used in this research covers the period 02 January 2012 to 31 March 2022. The returns on the CM are proxied by an index aggregating BTC and Ether (ETH). The market capitalization of these two cryptocurrencies represents more than 60% of the CM for the first

[2] The conclusion would be similar for a comparison between the log-returns on the CM and the two other variables.

Figure 1: Log-returns on the CM and the yield slope variation



Notes: This figure assesses the departure from the normality assumption in the log-returns on the CM and the first difference of the slope (interest rate spread) series. In level, the slope series is not stationary. We mostly work with the differenced series in the remainder of the paper.

quarter of 2022 (Cryptocompare 2022). Regarding the yield spread, it is computed for the US government bond market. Not only the latter is the largest of such markets, but also US treasuries are held by investors across the globe. The reasoning underlying the choice of the VIX and the 5-year breakeven inflation is also driven by the preeminence of the US financial system. Implicitly, the US yield slope, the VIX and the breakeven are the best candidates to gauge investors' possible trade-off with the CM.

Our first set of results shows no evidence of dependence between the CM and the government bond market. In particular, there is no revealed pattern for returns on the CM to cluster with a particular quantile of the interest rate spread. We illustrate this result through the plot of the marginal distributions of both variables in Figure 4. The low estimates of both the dependence and the tail parameters in Table 4 support the graphical representation of the two variables. So, investors on the CM give no weigh to expected policy paths and uncertainty about economic cycles in their decisions related to the CM as measured by the yield curve slope.

The dependence between volatility on the CM and the VIX exhibits a more complex picture. We find evidence for extreme low VIX values (5% quantile) to be correlated with high predicted volatility (90% quantile) on the CM (see Figure 5). We transform the VIX variable (100 minus the VIX) and express this observation in terms of an upper tail dependence structure, which has a known mathematical form in the copula theory. The likelihood of observing the cluster of these extreme events is estimated at 4.6% and 7.8% for the Gumbel-Hougaard and the Joe copulas respectively. Given the VIX is a 1-month ahead estimate, we generate a similar CM volatility measure and re-evaluate the tail dependence probability. The upper tail dependence estimates jump to 8% and 12%. Based on the last results, market participants should expect overlapping between low VIX and high CM volatility values every 13 and 9 days respectively, depending on which of the two copula families is considered.

We also find weak and non statistically significant estimates for the copula modelling exercise between the log-returns on the CM and the breakeven inflation. Overall, the results of the copula technique remains coherent in presence of possible nominal and real economic drivers of prices on the CM.

To ensure the validity of our results, we conduct various robustness checks in subsection 4.6. The first step involves a sub-sample analysis ranging from 1 January 2020 to 30 March 2022. This period witnessed the arrival of institutional investors and a relative recognition of CM activities from regulators. Surprisingly, our conclusions for the sub-sample also find no co-movement between returns on the CM, the interest rate spread and the breakeven inflation. The consistency of the results would mean that the arrival of smart money does not enhance the connection between cryptocurrencies and the wider macroeconomic environment. We obtain similar results when transforming the variables into weekly observations in order to control for possible non-synchronous closing times problem that has been documented to be a major setback in the the analysis of crypto-currencies (see Alexander & Dakos (2020)).

Slow-updating of investors' risk perception with information in the interest rate spread may be an important factor in the absence of dependence found in this research. In this scenario, models that can capture the delayed response of returns on the CM to change in the other variables can be insightful for the present analysis. We build a simple Autoregressive Distribute Lag (ADL) model to elucidate this point, where the interest rate spread and the breakeven inflation enter the equation as covariates. Again, the interest rate spread has no predictive power on the returns on the CM. On the contrary, a 1 percentage point increase in today's expected inflation corresponds to a 5.6 points increase in tomorrow's returns on the CM. This marginal effect is stronger (12%) and contemporaneous in the sub-sample 2020-2022.

In terms of economic knowledge, we provide evidence that crypto-currencies are weakly linked to some financial fundamentals, in particular uncertainty information contained in the interest rate spread and the VIX. Unlike the copula technique, our simple time series model uncovers a positively significant relationship between the breakeven inflation and the log-returns on the CM. The high proportion of ties in the interest rate spread (84%) and the breakeven inflation (94%) is a potential source of problems for the copula estimates. This issue may undermine the continuity assumption made by the Sklar's theorem on the marginal distribution of the mentioned series and affect the reliability of the copula estimates (see Hofert et al. (2019)). Going forward, a combination of copula and dynamic time series would be an efficient approach to explain price movements on the CM.

2 Related Literature

A starting point of this investigation is the literature on rational expectations and the term structure of interest rates. In particular, our contribution extends the long standing debate of the linkages between interest rates and risky assets into the CM literature. This work's methodology follows the approach in Estrella & Mishkin (1996) for the spread choice. In terms of early findings, numerous publications argue the existence of a significant link between the term structure of interest rates and economic activity (see e.g., Mishkin 1990, Estrella & Hardouvelis 1991, Ang et al. 2006). Work by Zhou (1996) and Boudoukh et al. (1997) also depict an important relation between interest rates on US government securities and equity returns. However, other work in the field cast doubt on the predictive power and the use of the yield slope in predicting future economic trends (see e.g., Shiller et al. 1983, Campbell & Shiller 1991). Our analysis applies the copula framework to both the entire sample and sub-samples of the dataset to detect possible interlinkages between the spread (the VIX and the breakeven inflation also) and returns on the CM. As a statistical tool, the copula technique lays out a straightforward approach to test this relationship while keeping the core theoretical underpinnings of the expectations theory intact.

This research adds to the empirical literature researching the linkages between crypto assets, economic policies and the traditional class of financial securities. Earlier studies have found significant impacts of monetary policy decisions on cryptocurrency price valuations. Focusing on BTC alone, Karau (2021) uncovers a strong connection between monetary policy stances and the BTC price. Corbet et al. (2020) also observe similar links between monetary decisions and cryptocurrencies. On a different approach, but closely related to our analysis, Akyildirim et al. (2020) pinpoint the existence of a correlation between cryptocurrencies and uncertainty on the stock markets, proxied by implied volatility measures. Our analysis offers an integrated investigation of the dependence between the CM, the risky and the risk-free market. Compared to the findings reported in this paragraph, our copula analysis finds no substantial relationship between the CM, the interest rate spread, the VIX and the breakeven inflation. The ADL model presents a nuanced picture with strong and statistically significant effects of the breakeven inflation on the log-returns on the CM. In a nutshell, these results make a case for the use of dynamic models (models with lags) in the analysis of crypto prices.

From a broader perspective, our research offers practical insights into price movements on the CM. A pioneered thinking on this question is from Böhme et al. (2015), who see the bitcoin money growth model as an inherent cause for the shallow market issue. Given the widespread use of cryptocurrencies for financial trading purposes, publications on price fluctuations on the CM have expanded largely over the recent years. Regarding the stylized facts, Zhang et al. (2018) analyse the returns of 8 leading cryptocurrencies and detect the existence of heavy tails, a pattern towards long memory, and a powerful feature of volatility clustering. Similarly, Hu et al. (2019) find a significant dissimilarity in the returns distribution for a sample of over 200 virtual currencies, which would probably indicate some restraints in generalising findings for a class of cryptocurrencies to the entire CM. These well-established statistical facts are supported in numerous volatility modelling publications (see also Bariviera 2017, Jiang et al. 2018). Our research finds the existence of persistent conditional volatility on the CM and shared properties with traditional financial time series. Unlike Böhme et al. (2015), we have not identified the monetary structure of BTC and ETH to be a driving factor in their price variations.

3 Data and Summary Statistics

3.1 Data construction methodology

We obtain statistics on cryptocurrencies, the spread on US government instruments, and the VIX from Bloomberg and the Federal Reserve Bank of Saint Louis respectively. BTC and ETH prices are observed daily at 5:00 PM, Eastern Time. The Exchange Rate Index (ERI) for the CM is made up BTC and ETH. As of 1 April 2022, BTC and ETH account for 61% of the overall market capitalization of the CM (see CoinMarketCap 2022). So, these two cryptocurrencies are actually representative of the crypto exchange activities.

Observations for BTC/USD are available from 02 January 2012 to 31 March 2022. However, data for ETH/USD are accessible from 08 February 2018 to 31 March 2022. To compute an index of the two rates at a given time t, the Dow Jones methodology is implemented as follows $\frac{P_{1t}+P_{2t}}{n}$, where P_{1t}, P_{2t} and n stand for the exchange rate of BTC, the exchange rate of ETH and the number of price series respectively. From 02 January 2012 to 07 February 2018, the ERI is simply equal to the price of BTC in USD. Although ETH trading activities went back to 2015, the Bloomberg ETH price series starts in February 2018. Noise affecting ETH price in the early trading days might be a reason for this choice from Bloomberg. To smooth the introduction of ETH in the index calculation, a divisor is used to compute the ERI from 08 February 2018 till the end of the series. The divisor is calculated as the summation of the prices divided by the previous day index $(\frac{P_{t1}+P_{t2}}{ERI_{t-1}})$. Instead of the number of cryptocurrencies, the sum of the two cryptocurrency prices is divided by the divisor in the modified formula $(ERI_t = \frac{P_{1t}+P_{2t}}{Divisor})$. We subsequently refer to the variation of the index as the cryptocurrency returns or simply X_{1t} in the next section.

The study relies on the interest rate spread on US government bonds to offer a comprehensive analysis of the CM. The spread used in this work is the difference between the interest rates on the 10-year treasury note and the 3-month treasury bill. Estrella & Mishkin (1996) provide a thorough empirical analysis of the strength of the interest rate spread considered here in predicting macroeconomic cycles. More specifically, the conclusion of their investigation shows a relatively important long-term prediction capability of the interest rate spread between the 10-year note and the 3-month bill. We use a similar motivation to study how price movements on the CM can be approximated by information contained in the interest rate spread. Mathematically, The yield on a zero-coupon government bond with a 1 US dollar face value is defined as $y_t = \left(\frac{1}{P_0(0,t)}\right)^{\frac{1}{t}} - 1$, with $P_0(0,t)$ describing the price of the bond quoted (and purchased) at time 0 and expiring in t periods. Formally, our spread variable is defined as $X_{2t} = y_{10} - y_{0.25}$.

The empirical section also encompasses two uncertainty and forward measures. First, the VIX measures the expected volatility regarding the S&P500. Second, the 5-year breakeven inflation gives the market expectation of the average inflation for a 5-year horizon. The co-movement analysis of these variable with returns on the CM would give us valuable information regarding market participants on the CM.

Overall, the dataset at hand contains 2674 observations. The latter are sampled on business days only. Regarding missing values, the identified cases are filled up according to the Kalman filter approach. As opposed to linear interpolation or related techniques, the latter is preferred due to its ability to replace missing values while preserving the existing trend or seasonal pattern observed in the series.

In the following sections, we refer to returns on the CM, the yield slope, the VIX, the 5-year breakeven inflation and any transformation arising from them as X_{1t}, X_{2t}, X_{3t} and X_{4t} respectively.

3.2 Summary Statistics

Table 1 reports descriptive statistics of the main variables used in this research. We compute daily returns on the CM as $X_{1t} = log(\frac{ERI_t}{ERI_{t-1}})$. Returns on the CM are left-skewed and characterized by an excess kurtosis. Average returns for the sample period oscillate around 0.003 for the CM. Note that the log-returns distribution range from -0.6 to 0.52. X_{1t} range conveys evidence of extreme fluctuations in cryptocurrency trading activities. In concrete terms, daily log-returns on the CM have been alternating between -60% and 52% over the past 10 years. In contrast, the first difference of the interest rate spread (slope of the yield curve), denoted X_{2t}, has shown less variation. The average value is close to 0 with a skewness of 0.47. Overall, the slope of the yield curve has been mostly positive for US government bonds with maturities set for 3 months and 10 years.

Table 1: Summary statistics of the main variables

Variable	N	Mean	Std. Dev.	Min	Skewness	Kurtosis	Max
X_{1t}	2,673	0.003	0.053	-0.60	-0.60	25	0.52
X_{2t}	2,673	5.59×10^{-7}	0.0004	-0.002	0.47	6.25	0.003
X_{3t}	2,673	17.27	6.85	9.14	3.18	20.65	82.69
X_{4t}	2,673	0.00001	0.0004	-0.003	-0.0034	0.0022	0.002

Notes: This table reports basic statistics for log-returns on the CM, change in the spread series, and the VIX.

The log of ERI shows an increasing trend throughout the entire sample. Compared to 02 January 2012, the index was multiplied by more than 8000 on 31 March 2022. As evidenced by the top-right panel of Figure 2, volatility clustering has been persistent for the period covered in this research. The two most important volatility bursts in the log-returns of ERI occurred around April and December 2013. The first one emerged from news of liquidity issues faced by two pioneered crypto exchanges (BitInstant and Mt. Gox) to meet their obligations towards their investors. As a result, the price of bitcoin nosedived 60% on 11 April 2013 before realizing a rebound of 32% seven days later. The second corresponded to the warning issued by the People's Bank of China on 05 December 2013 against the use of BTC in financial transactions. BTC price declined by 58% following the announcement and regained much of its value on 09 December 2013 (52%). It is worth mentioning that the introduction of ETH in the sample (purple vertical line) does not lead to any abnormal change in the fluctuations observed on the CM. The observation also goes for the vertical red line, which indicates the true inception date of ETH on 30 July 2015.

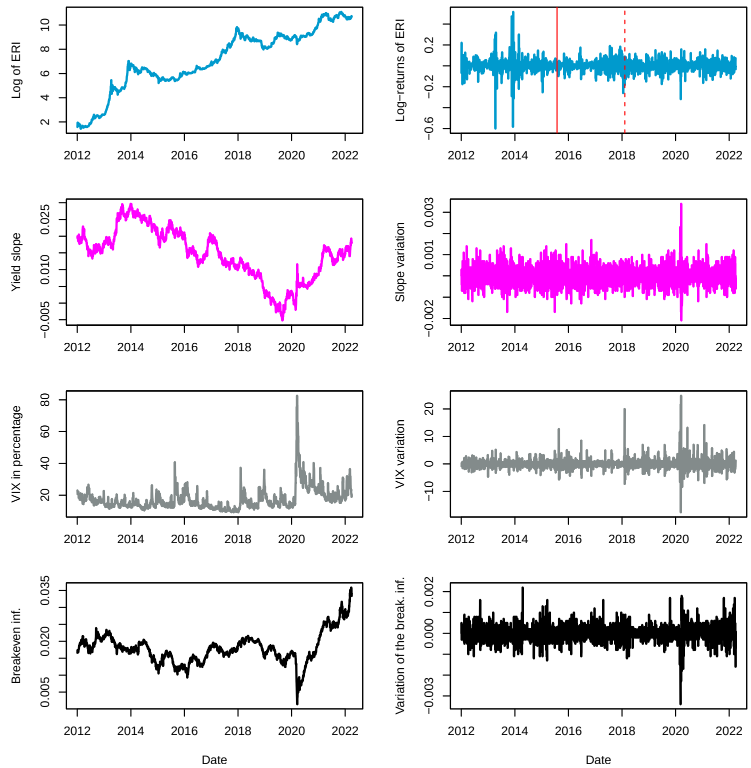
In the volatility modelling process, the emphasis is on the change of the spread series, meaning the first difference (X_{2t}). In fact, the spread series, the second row of Figure 2, is non-stationary in level [3]. So, the change in the spread, which is stationary, is important for the conditional copula modelling process. The latter uses the Generalized Autoreregressive Conditional Heteroskedasticity (GARCH) modelling framework as an input to compute dependence parameters, making the stationarity of the margins a critical element. Furthermore, the stationarity of the margins are important to ensure the application of the copula diagnostic tests (Hofert et al. 2019). In terms of economic interpretation, X_{2t} captures similar information as the level series, meaning investors attitude towards future change in the economy. However, evidence of volatility clustering seems to be less dominant in X_{2t} in comparison to X_{1t}. Aside a few episodes of peaks and drops, fluctuations in X_{2t} are constrained within -0.1% and 0.1%

Inspecting the logarithmic of ERI and the spread in level shows a divergent trend between 2014 and 2020. As the spread between long-term and short-term interest rates was shrinking, cryptocurrencies prices were becoming more important. Yet, this makes a compelling case for the use of the first difference of the spread series. Successive reductions of the spread between the two interest rates turn into negative values in the first difference transformation. The negative signs in X_{2t} will come in handy when studying co-movement with X_{1t}. So, existence of negative values for both X_{1t} and X_{2t} at comparable time periods would be important aspects for the tail dependence in the bivariate copula analysis.

The last two variables of Table 1 and Figure 2 are the VIX and the breakeven inflation. The most important observation is the synchronization of volatility movements with returns on the CM around early 2021. We offer a sub-sample analysis in the next section that sheds light on the relative co-movement detected in these plots.

[3] See the Autocorrelation Function (ACF) in Figure 9 of the appendix for an analysis of each series' departure from the stationarity assumption

Figure 2: Plot of the variables in level and their first difference transformations



Notes: This figure presents the four main variables of the study. The red line illustrates the effective issuance date of ETH (30 July 2015), whereas the purple one corresponds to the introduction period of ETH in the present sample (08 February 2018).

4 Model Specification and Results

4.1 Theoretical Motivation

As explained in the introduction, this study is built on the assumption of a trade-off between cryptocurrencies and traditional financial instruments. We start with the intuition of a possible negative correlation between cryptocurrencies and the latter class of assets. The reasoning underlying the relationship hinges on the expectations theory. For instance, a positive spread or an upward sloping curve is interpreted as a signal of future short-term interest rate hikes or economic expansion. We would expect investors on the CM to capture these signals of possible higher rewards from the wider financial market and opt for safer assets (bonds). The resulting outflow of capital from the CM would induce a negative relationship between the two markets.

Our second focus is the VIX and the breakeven inflation. The two variables allow us to expand our analysis of the CM beyond the classical trade-off between risky and risk-free assets. More specifically, we look at the co-movement between the CM, the stock market and the expected inflation.

4.2 Overview of the Copula Theory

A bivariate copula modelling implies finding parameter estimates for each variable and the dependence between them. The process starts with classical probability descriptions for continuous random variables. For instance, a 2-dimension random vector $\mathbf{X} = (X_{1t}, X_{2t})$ can be defined by its joint Cumulative Distribution Function (CDF) noted $H(x_1, x_2) = P(X_{1t} \leq x_1, X_{2t} \leq x_2)$ or in terms of the respective margins $F_1(x_1) = P(X_{1t} \leq x_1)$ and $F_2(x_2) = P(X_{2t} \leq x_2)$.[4] The Sklar's theorem states that $H(x_1, x_2)$ can be transformed into a function C, denoted copula, giving information on both margins and the dependence between the two variables. Formally, Sklar's theorem stipulates that:

$$H(x_1, x_2) = C(F_1(x_1), F_2(x_2)), (x_1, x_2) \in R^2. \tag{1}$$

A useful transformation involves applying the integral transform theorem on the component of each margin to make the arguments of $H(.)$ uniformly distributed over the interval $[0, 1]$. So, the copula function can now be written in terms of uniform components as:

$$C(u_1, u_2) = P(F_1(X_{t1}) \leq u_1, F_2(X_{t2}) \leq u_2). \tag{2}$$

Statistical properties underlying the importance of copula are subject to a relatively dense literature. One key element of the copula pertains to its role in detecting complex dependence structure between random variables. For instance, contrary to a parametric dependence measure such as the simple correlation that is restricted to two variables (which should be linearly linked), copula can be generalized for any k-dimension vector of random variables. Hence, a k-dimension copula will simple be written as

$$C(x_1, x_2 ..., x_k) = (F_1(x_1), F_2(x_2), ..., F_k(x_k)), \tag{3}$$

which is a mapping of $[0, 1]^k \rightarrow [0, 1]$.

So far, the account presented in this segment touches upon a basic definition of the copula framework. However, numerous variants of copulas have been developed in the recent literature. Table 2 presents the main copula families encounter in empirical work in finance.[5] The level of the θ parameter controls the dependence between the random variables at hand. In fact, a crucial

[4] The mathematical notation in this section and the following ones are based on Nelsen (2003), Mikosch (2006) and Hofert et al. (2019).

[5] Elliptical copulas (Normal and Student's t) and Archimedian copulas (Clayton, Gumbel, Joe and Frank) are among the most used in financial studies. McNeil et al. (2015) and Hofert et al. (2019) explore in great detail other techniques, such as copulas related to the extreme value theorem.

difference among families of copulas lie in the notion of tail dependence. The latter is a conditional probability that measures the likelihood of both X_1 and X_2 facing an extreme event (or lie above/below a certain quantile denote q). After setting up a given quantile, the lower (τ^L) and upper tail (τ^U) are given by

$$\tau^L = \lim_{q \to 0^+} P[(X_{t1} < F_1^{-1}(q) | X_{t2} < F_2^{-1}(q)] = \lim_{q \to 0^+} \frac{C(q,q)}{q}$$

$$\tau^U = \lim_{q \to 1^-} P[(X_{t1} > F_1^{-1}(q) | X_{t2} > F_2^{-1}(q)] = \lim_{q \to 0^+} \frac{1-2q+C(q,q)}{q}.$$

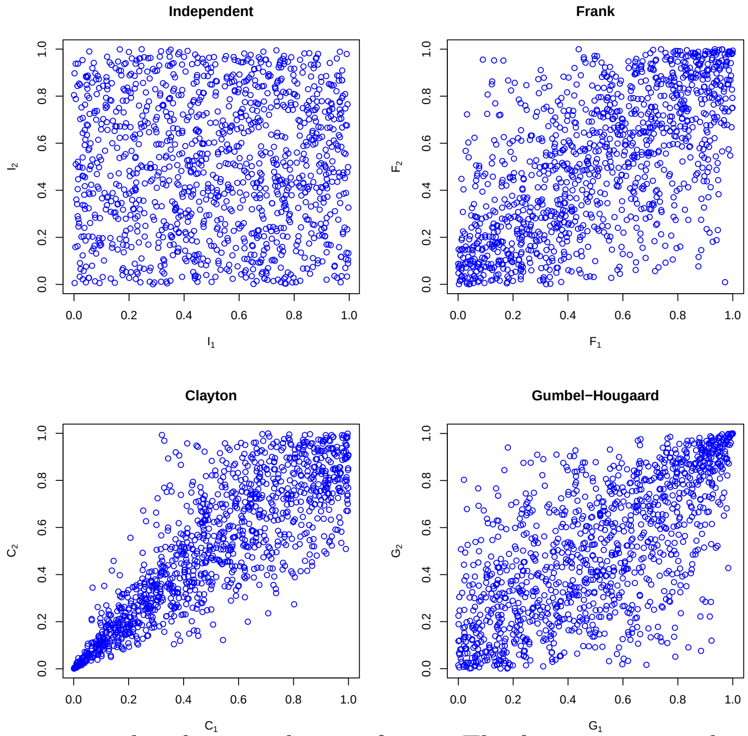
Table 2: Summary of some of the widely used copula families

Type	**Parameter (θ)**	$\boldsymbol{C(u_1, u_2)}$	$\boldsymbol{\tau^L}$	$\boldsymbol{\tau^U}$
Normal	[-1,1]	$N_\theta(\phi^{-1}(u_1), \phi^{-1}(u_2))$	0	0
Student's t	[-1,1]	$t_{\theta,v}\,(t_v^{-1}(u_1), t_v^{-1}(u_2))$	$2t_{v+1(w)}$	$2t_{v+1(w)}$
Clayton	$(0, \infty)$	$(u_1^{-\theta} + u_2^{-\theta} - 1)^{\frac{-1}{\theta}}$	$2^{\frac{-1}{\theta}}$	0
Frank	$(0, \infty)$	$\frac{1}{\theta} log\left[1 + \frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{(e^{-\theta}-1)}\right]$	0	0
Gumbel-Hougaard	$[1, \infty)$	$exp\left[-\left(\left(log u_1\right)^\theta + \left(log u_2\right)^\theta\right)^{\frac{1}{\theta}}\right]$	0	$2 - 2^{\frac{1}{\theta}}$
Joe	$[1, \infty)$	$1 - \left[\left(1-u_1\right)^\theta + \left(1-u_2\right)^\theta - \left(1-u_1\right)^\theta \left(1-u_2\right)^\theta\right]^{\frac{1}{\theta}}$	0	$2 - 2^{\frac{1}{\theta}}$

Notes: In-depth explanation is provided in (McNeil et al. 2015) and the references in the theoretical section. The tail dependence of the Student's t is obtained as the CDF of a univariate distribution with $v+1$ degrees of freedom, with $w = \frac{-\sqrt{v+1}\sqrt{1-\rho}}{\sqrt{1+\rho}}$. ϕ and N are denoted CDF of a univariate standard normal distribution and CDF of a bivariate normal distribution respectively.

Unconditional and conditional estimations of copula parameters are conducted in section 4. The choice of the suitable copula technique is supported by a mix of graphical and goodness-of-fit tests. For illustration purposes, four copula representations are generated in Figure 3. The Frank, the Clayton and the Gumbel-Hougaard replicate the graphical pattern in table Table 2 in terms of tail dependence (concentration of observations either near the point $(0, 0)$ or $(1, 1)$, which are situations of lower and upper tail dependence respectively). Thus, a graphical analysis contains a paramount role in copula evaluation.

Figure 3: Graphical representation of four simulated bivariate copula families



Notes: Each figure is generated with a sample size of 1200. The θ parameter is arbitrarily set to 5 for Frank, 4 for Clayton and 2 for Gumbel-Hougaard.

4.3 Cryptocurrency Market and Investors' Expectation

Similar to any classical estimation framework, working with copula consists in estimating the parameter θ of the copulas reported in Table 2 and identifying the best approach to model the joint distribution. When dealing with time series, one common approach is to estimate the parameters of $C(.)$ parametrically using the Maximum Likelihood Estimation (MLE). This has the advantage of handling both the marginal and the copula dependence parameters.

Copula estimates are normally computed in multiple stages. The first step entails formulating a stochastic process driving returns on the CM and the change in the spread. Then, the standardized residuals are used to estimate the dependence between the two variables. The conditional copula specification for returns on the CM and the change in the spread involves a simple mathematical twist with regard to the case in the previous section known as unconditional copula. Again, using Hofert et al. (2019) notations, the conditional form of the copula is:

$$H_{\mathcal{G}t-1}(x_1, x_2) = P(X_{1t} \leq x_1, X_{2t} \leq x_2 | \mathcal{G}_{t-1}), (x_1, x_2) \in R^2. \tag{4}$$

In this set up, $\mathcal{G}t - 1$ is the information set incorporating past indications about returns on the CM, the change in the spread and their dependence. Again, Sklar's theorem allows to write the joint conditional distribution to be formulated in terms of conditional copula as

$$H_{\mathcal{G}t-1}(x_1, x_2) = C_{\mathcal{G}t-1}(F_{\mathcal{G}t-1,1}, (x_1), F_{\mathcal{G}t-1,2}, (x_2)). \tag{5}$$

In practice, the conditional transformation of equation 4 involves using the standardized residuals from an ARMA-GARCH structure to estimate the dependence parameter in the copula notation. In this work, conditional components (mean and variance) of returns on the CM and the change in the yield are of the form ARMA(0,0,0)-GARCH(1,1).

Equations 6 and 7 display the conditional mean and the conditional equations for the two markets. The ACF and the additional analysis in Appendix B provides justifications for the autoregressive order followed in the specification below. We also use the stepwise algorithm to confirm the intuition behind the interpretation of the ACF [6]. The two sets of equations take the form

$$\begin{aligned} X_{1t} &= \mu_{1t} + \epsilon_{1t} \\ \delta_{1t}^2 &= \omega + \alpha_1 \epsilon_{1t-1}^2 + \beta_1 \delta_{1t-1}^2 \\ \epsilon_{1t} &= \delta_{1t} e_{1t} \\ e_{1t} &\overset{iid}{\sim} t_v(0,1) \end{aligned} \tag{6}$$

$$\begin{aligned} X_{2t} &= \mu_{1t} + \epsilon_{2t} \\ \delta_{2t}^2 &= \omega + \alpha_1 \epsilon_{2t-1}^2 + \beta_1 \delta_{2t-1}^2 \\ \epsilon_{2t} &= \delta_{2t} e_{2t} \\ e_{2t} &\overset{iid}{\sim} N(0,1), \end{aligned} \tag{7}$$

The assumption made on the distribution of the standardized residuals is the main difference between equations 5 and 6. e_{1t} follows a skewed student's t process whereas e_{2t} is normal distributed. Figure 12 in the appendix section compares the fitness of equations 5 and 6 with normal, student's t, skewed student's t and generalized error residuals. The GARCH(1,1) with skewed residuals stands out as a good candidate to explain price variations on the CM. Similarly, the GARCH(1,1) with normal residuals is the best option for the spread series.

Parameters of equations 5 and 6 are estimated from the joint density function of $H(.)$ using the MLE. The likelihood notation takes the form of a joint density product of the marginal densities

[6] The stepwise regression gives evidence for an ARMA(0,0,0) with no conditional mean in the case of the spread equation. This situation arises because the conditional mean is relatively low and statistically not different from 0. We depart from the suggestion of the stepwise algorithm and estimate the conditional mean (the slope). This deviation does not change the conclusion drawn in this chapter.

and the copula density

$$f(\mathbf{X};\Omega,\psi) = f_1(X_{1t};\Omega_1)f_2(X_{2t};\Omega_2)c(u_1,u_2;\psi). \tag{8}$$

We apply the two-stage approach by first estimating all margin parameters in Ω_1 and Ω_2. Degrees of freedom and a shape parameter for X_{1t} residuals are also computed as part of the first stage, since shocks in this particular model are skewed student's t distributed. The vector of copula parameters (ψ) are again computed via the MLE in the second stage. These two steps are visible from the log likelihood of the joint density, where the sum of the marginal log likelihoods and the copula log likelihood form the first and the second stage respectively. The log-likelihood expression is written as

$$\mathcal{L}f(\Omega,\psi;X) = \ln f_1(X_{1t};\Omega_1) + \ln f_2(X_{2t};\Omega_2) + \ln c(u_1,u_2;\psi). \tag{9}$$

Note the expressions 2 and 9 write the copula function in terms of the uniform margins u_1 and u_2. This transformation is crucial to the estimation of the copula parameters. The process requires extracting the standardized residuals of the ARMA-GARCH processes and apply the integral transform theorem in order to obtain the uniform margins from the empirical distribution of the residuals. In this paper, we follow the recommendation in Hofert et al. (2019), where the uniform margins in the copula density are estimated by:

$$U_{it} = \frac{1}{n+1}(R_{it}), \tag{10}$$

with R_{it}, the rank of a residual observation in the dataset. The position or the rank is determined by the time index t. The U_{it} sample is known as pseudo-observations in the literature.

Estimates of the marginal series show significant volatility persistence with the sum of α_1 and β_1 being close to 1. The persistence is, however, much higher in the case of the CM, which unequivocally subscribes to the description of volatility clustering. According to the half-life calculation, it takes 346 days for the volatility to revert to 50% of its long term level following a shock on the CM. Standardized residuals or shocks on the CM are slightly leptokurtic and positively skewed as shown by the estimates of the shape parameters. Note that both the conditional mean and variance of X_{2t} are not statistically different from zero. This would indicate that past information is irrelevant to understand fluctuation in the spread series. The estimated conditional moments of the spread series stand in contrast to the CM, although the conditional variance is relatively small in the case of the cryptocurrency.

Table 4 reports the dependence parameter estimates for a set of copula families widely used in empirical finance. In exception of the clayton copula, the remainder of the estimates is not statistically significant at the conventional significance levels. The relatively large degree of freedom (29) is evidence that a normal copula would be preferred to the student's t family in the context of this study. By the same token, the low value of the dependence parameter indicates weak dependence between fluctuations on the CM and the term structure. The computed tail probabilities are approximately zero for the different copula types. So, observed extreme events on the two markets are likely to be unrelated.

Table 3: ARMA-GARCH estimates of returns on the CM and the treasury yield spread

	X_{1t}	X_{2t}
	ARMA(0,0)-GARCH(1,1)	ARMA(0,0)-GARCH(1,1)
μ	0.003^{***}	1×10^{-6}
	(0.001)	(8×10^{-5})
ω	$5.5\times10^{-4***}$	1.2×1^{-7}
	(1.8×10^{-4})	(2×10^{-7})
α_1	0.159^{***}	0.073^{***}
	(0.019)	(0.016)
β_1	0.840^{***}	0.901^{***}
	(0.020)	(0.014)
v	0.998^{***}	
	(0.022)	
λ	3.068^{***}	
	(0.147)	
Log. Likelihood	4,934.416	16,937.91
AIC	−3.688	-12.670
N. obs.	2673	2673

Notes: In this table, v and λ are estimates of the degrees of freedom and the skewness parameter in a GARCH model with Hansen's skew t error. In parenthesis are the standard errors of the estimates.
***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level

We plot the uniform transformed components of the residual series in Figure 4. In accordance with the copula estimates, no clear dependence pattern is observable between the CM and the US government bond market. The graph rather illustrates the case of an independence copula. In theory, a clayton parameter estimate oscillating around zero or a gumbel parameter around 1 is a sign of independence copula. We formally test the null hypothesis that the relation between the two markets is no different from an independence copula structure [7]. Unsurprisingly, we find no evidence against the null hypothesis (p-value=0.226). To ensure the result of this analysis is not a mere consequence of the copula specification, we test for the necessity of a time-varying copula modelling using the Bücher et al. (2014) method. The latter accounts for deviations in the dependence parameter due to the changing nature of the dependence between the two margins or abrupt structural breaks in the two series. We find strong evidence against a time-varying copula

[7] This test evaluates whether the Kendall's rank correlation is statistically different from zero or not. The correlation level is 0.015, which is not different from zero according to the test. As a side note, this test is possible because rank-based correlation parameter can be written as a function of the underlying copula between the two variables.

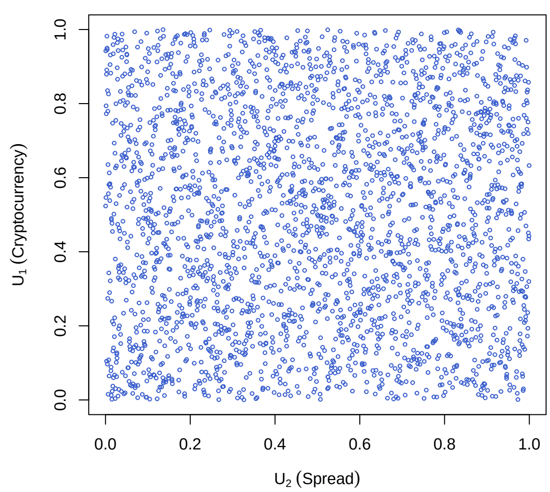
Table 4: Estimates of different copula family parameters

	Normal	Student's t	Clayton	Frank	Gumbel-Hougaard	Joe
θ	0.022	0.024	0.049**	0.156	1.001	1.003
	(0.019)	(0.02)	(0.02)	(0.114)	(0.011)	(0.016)
τ^L		8.252×10^{-6}	$7.488\times10^{-7***}$			
		(5.281×10^{-6})	(1.52×10^{-8})			
τ^U		8.252×10^{-6}			$2.065\times10^{-8***}$	$2.065\times10^{-8***}$
		(5.281×10^{-6})			(2.322×10^{-10})	(3.318×10^{-10})
Deg. of freedom		29				
		(19.23)				
Log. Likelihood	0.653	1.873	2.947	0.887	-2.749×10^{-7}	-1.091×10^{-6}
AIC	-0.695	-4.254	1.895	-0.226	-4	-4
N. obs.	2673	2673	2673	2673	2673	2673

Notes: In the spirit of Table 2, the statistical significance of the Joe and Gumbel-Hungaard is tested as $H_0 : \theta = 1$ and $H_1 : \theta > 1$. The standard errors for the tail probabilities, in parenthesis, are computed with the delta method, since the latter is a transformation of θ.
***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level .

(p-value = 0.373). This first set of results is indeed a robust illustration of the independence nature of the two markets modelled through the bivariate copula, which is the antipode of the rational expectation theory. In the sense that the slope of the yield curve for the selected maturities is not relevant in explaining fluctuations on the CM. We will later go back to the dynamic nature of the analysis and robustness considerations.

Figure 4: Dependence representation between the spread and the CM



Notes: This figure presents the residuals extracted from equations 5 and 6. The residuals are transformed to be distributed between 0 and 1 (Pseudo-observations). To operate the transformation, we use the rescaled empirical distribution function approach. Hofert et al. (2019) gives a detailed explanation of this technique.

4.4 Cryptocurrency Market and Volatility Anticipation

We shift our attention to studying the link between expected volatility on mainstream risky markets and price fluctuations on the CM in this section. The GARCH-based estimate of the crypto price volatility is used for the CM. As explained in the introduction, the S&P 500 is our proxy for the overall stock market. As such, we use the VIX to measure expected volatility regarding the global stock market. The VIX is often seen in the literature as a proxy for investors' sentiment and uncertainty about the future state of the equity market (see Bekaert et al. 2013). So, the existence of a significant correlation between the VIX and the volatility on the CM would help pinpoint possible fundamentals driving cryptocurrency price movements.

Unlike Bekaert et al. (2013) that breaks the VIX down into an uncertainty and a risk-aversion component, the estimations below use raw values of the VIX as downloaded from the Federal reserve Bank of Saint-Louis. Our interest is simply the VIX component in level. In other terms, we want to appraise how volatility on both markets relates to each other. A primary investigation shows that high volatility on the CM tends to synchronize with low volatility expectations on the S&P 500 (See top-left panel of Figure 5). To better capture the latter observation and compute a tail dependence probability, we reverse the distribution of the VIX by subtracting the index from 100. The synchronization is now translated into an upper tail dependence representation, which is computed in the last column of Table 5 and visible in the top-right panel of Figure 5.

We present semiparametric copula estimates for the equity market and the CM in Table 5. In the previous section, we conduct the dependence estimation work using the residuals of the GARCH processes. We used the residuals to account for the volatility clustering feature of the returns series in the copula parameter estimation. In this section, we directly generate the pseudo-observations (with equation 10) using the values of the VIX in level and the GARCH-based volatility estimated from equation 6. Then, the copula dependence parameters are estimated between X_{1t} and X_{3t} via the MLE [8]. In line with the previous results, the dependence parameter estimates are relatively low and statistically different from zero for the different copula families. Unlike results presented in the previous section, the tail dependence probability is non-negligible (4.6%) for the Gumbel-Hougaard and the Joe copulae (7.8 %). So, if a trader were to combine the ERI and the S&P 500 index, she should expect the volatility on each market to go in different directions every 22 days with the Gumbel-Hougaard and every 13 days with the Joe. However, judging from the AIC, the Gumbel-Hougaard offers a better fit than the Joe Copula and would therefore be a stronger statistical framework to study the relationship between the variables (with the student's t copula being the best model).

[8] A semiparametric estimate avoids the steps of specifying a GARCH structure for the VIX. In the present analysis, the semiparametric choice does not alter the conclusion if we were to estimates the dependence coefficients parametrically (fully). In addition to the simplicity of the semiparametric approach, the GARCH estimates for a VIX series would be hard to make sense of (in the first estimation stage) as opposed to price series where the mean and variance equations have meaningful financial implications.

Table 5: CM and VIX copula estimates

	Normal	Student's t	Clayton	Frank	Gumbel-Hougaard	Joe
θ	0.062***	0.062***	0.062**	0.295***	1.031***	1.061***
	(0.02)	(0.02)	(0.027)	(0.119)	(0.013)	(0.021)
τ^L		0.000	1.401×10^{-5}***			
		(1.56×10^{-10})	(3.778×10^{-7})			
τ^U		0.000			0.046***	0.078***
		(1.56×10^{-10})			(0.009)	(0.002)
Deg. of freedom		3847***				
		(7.171)				
Log. Likelihood	5.047	5.045	-	3.287	4.455	6.006
AIC	8.095	2.091	-	4.574	4.910	8.013
N. obs.	2673	2673	2673	2673	2673	2673

Notes: This table presents estimates of some of the widely used copulas. In parentheses are standard errors of the estimates. In the spirit of Table 2, the statistical significance of the Joe and Gumbel-Hungaard is tested as $H_0 : \theta = 1$ and $H_1 : \theta > 1$. The standard errors for the tail probabilities are computed with the delta method. The MLE failed to estimate the dependence parameter in the Clayton case. We instead estimate the parameter via the method-of-moment (Spearman's rho). Hofert et al. (2019) gives a detailed explanation of this technique.
***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level

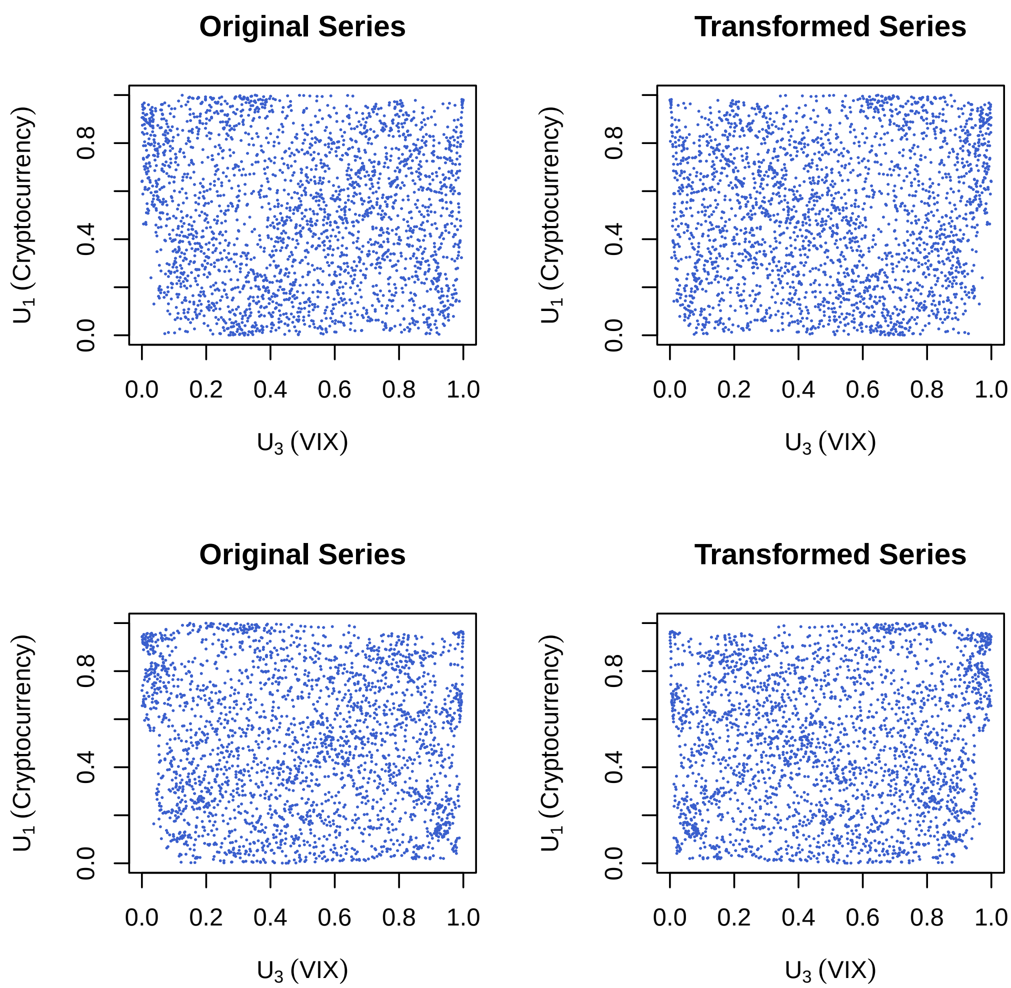
The margins of the two variables are plotted in Figure 5. As explained above, the first column gives a representation of the VIX along with the volatility on the CM. The second column, labelled as transformed series, flips the cluster of observations near the point $(0, 1)$ to the point (1,1). So, the Gumbel-Hougaard and the Joe upper tail dependence probabilities output the odds of having this cluster of points.

We also display a representation of the VIX along with a GARCH-based forward volatility estimate for the CM in the bottom panels of Figure 5. Given the VIX is a forward-looking variable, investors' forward volatility expectation for the CM may show a stronger response to change in the VIX than the instantaneous volatility analysis conducted in the previous paragraphs. The forward volatility, computed as $\frac{1}{22}\sum_{k=1}^{22}\sigma_{t+k}$, is a rolling ahead moving average over 22 trading days (or one calendar month), with k being the one period ahead index. In terms of level, the new dependence parameter estimate is close to the results of Table 5. [9] However, we find a relatively important difference in the likelihood of observing joint extreme movements on both markets. For instance, the tail dependence probabilities are now 8% and 12% for the Gumbel-Hougaard and the

[9] One of the first steps in this section was to establish the stationarity of the variables being used. We use a version of the test for point detection to clarify this point (see the section 6.2.1 of Hofert et al. (2019)). We find no evidence to reject the hypothesis of stationarity for the GARCH-based volatility (p-value=0.271). The 22-day moving average for the GARCH-based volatility is also stationary (p-value=0.284). We cannot reject the stationarity hypothesis for the VIX series at 1% significance level (p-value=0.026). For simplicity, we go ahead and accept stationarity for both variables.

Joe techniques respectively (see Table 8 in the appendix).

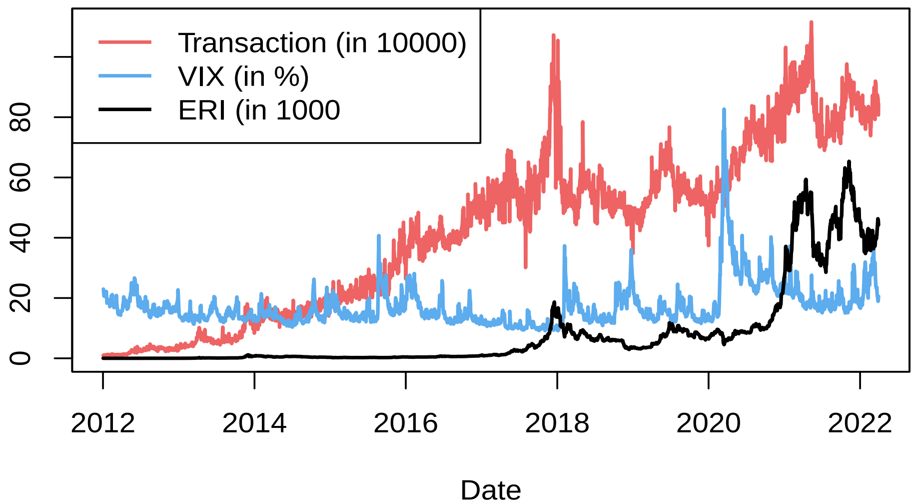
Figure 5: Dependence representation between the VIX and CM volatility



Notes: This figure presents the VIX and the volatility on the CM. The first row gives the VIX and the predicted volatility at time t. The second row is gives the VIX and the forward predicted volatility over 22 trading days. The values are transformed to be distributed over the interval 0 and 1. The original series plot the pseudo-observations with no transformation. However, the plots with the transformed series come from subtracting 100 from the VIX.

The relatively weak dependence of this section showcases the fact that the CM and the global stock market may value expected uncertainty in different ways, which leads to different price reactions on both markets. Relying on the leverage effect, meaning the existence of a negative correlation between volatility and returns, one possible explanation for the results would be the change expected low volatility brought about future expected returns. This would mean expected low volatility on the S&P 500 renders the market more attractive (inflow of capital) and bids returns up. In such a context, risk-averse investors would prefer the S&P 500 to the riskier CM, which would drive transaction activities down and increase fluctuations on the CM. This outcome would be a perfect illustration of the risk shifting mechanism and its resulting impact on returns distribution on the CM. However, data on transactions activity for BTC and ETH do not support this line of reasoning. For instance, Figure 6 does not show any clear pattern for transactions volume (activity index) within the BTC and ETH network to lower when the VIX weakens. The same lack of co-movement is also observed between the CM price index and the VIX.

Figure 6: Transactions in the cryptocurrency network, VIX and ERI



Notes: This figure presents the evolution of the VIX with respect to transactions in the cryptocurrency network (activity index) and the price index. Transactions activity is an index computed with the same methodology as ERI. Note that both the ERI and the transactions activity index are used in level.

4.5 Cryptocurrencies and inflation expectation

The conclusion reported in subsection 4.3 would be similar if interest rates on Treasury Inflation-Protected Securities (TIPS) were to be used rather than nominal ones. In fact, it is theoretically sound to assume investors care about real earnings and this fact should reflect in the correlation between the returns on the CM with interest rates on TIPS depending on the state of the inflation expectation (high or low). However, previous studies found no significant evidence for investors to hold more inflation-protected financial instruments when inflation expectations are high (Shiller 2015, Fleckenstein et al. 2014). We observe similar patterns in the dependence structure between crypto-currency price movements and the 5-year US breakeven inflation.

In the conditional copula framework, we model the breakeven inflation (first difference) as an ARMA(0,0)-GARCH(1,1) with GED residuals. Model checking and justifications for this GARCH order can be found in Figure 16. The copula estimates are reported in Table 6 and the scatter plot of the marginal distribution in Figure 17 of the appendix. Estimates of the copula and the tail dependence parameters are low and statistically not significant. The graph of the uniform-transformed margins shows no sign of tail dependence or relationship between the two variables. The Bücher et al. (2014) test for time-varying copula also provides no evidence against a constant conditional copula model (p-value=0.282). So, the results obtained in this section are not due to abrupt changes in the margins of either the log-returns of crypto-currencies or the breakeven inflation.

Table 6: CM and inflation expectation copula estimates

	Normal	Student's t	Clayton	Frank	Gumbel-Hougaard	Joe
θ	0.032	0.029	0.059	0.128	1.007	1.001
	(0.019)	(0.02)	(0.021)	(0.117)	(0.011)	(0.013)
τ^L		1.39×10^{-5}	8.37×10^{-6}			
		(0.017)	(1.743×10^{-7})			
τ^U		1.39×10^{-5}			0.009	2.066×10^{-8}
		(0.017)			(9.847×10^{-5})	(2.709×10^{-10})
Deg. of freedom		28*				
		(16.62)				
Log. Likelihood	1.349	2.879	4.674	-	0.194	-1.55×10^{-8}
AIC	0.699	-0.242	5.348		-3.612	-4
N. obs.	2673	2673	2673	2673	2673	2673

Notes: This table presents estimates of some of the widely used copulas. In parentheses are standard errors of the estimates. In the spirit of Table 2, the statistical significance of the Joe and Gumbel-Hungaard is tested as $H_0 : \theta = 1$ and $H_1 : \theta > 1$. The standard errors for the tail probabilities are computed with the delta method. The MLE failed to estimate the dependence parameter in the Frank case. We instead estimate the parameter via the method-of-moment. Hofert et al. (2019) gives a detailed explanation on the use of this technique.
***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level

4.6 Further Interpretation and Robustness Checks

Contrary to insights popularized in business and financial magazines, the results of this research fall short of establishing a significant correlation between the cryptocurrency market and the spread on the US government bonds [10]. We also find mild evidence linking the volatility on the CM with the expected volatility on the S&P 500 index, which is in a stark contrast with a similar study conducted by Akyildirim et al. (2020). The same weak evidence is also reported in the context of the breakeven inflation. This lack of connection would suggest that investors on the CM give little weigh to the slope of the yield curve, the VIX indicator and the expected inflation in their investment decisions. However, the first two variables are traditionally seen as strong predictors of

[10]This analysis by The Economist (2022) is one of these news articles explaining the April 2022 price drop by the rising interest rate on US government debt instruments.

change in financial conditions and business cycles (see e.g., Estrella & Mishkin 1996). Therefore, movements in the slope of the yield curve and the VIX contain valuable economic information for portfolio construction and investment decisions related to the mainstream markets. So, it is crucial to pinpoint elements that can explain the results found in the context of the CM analysis.

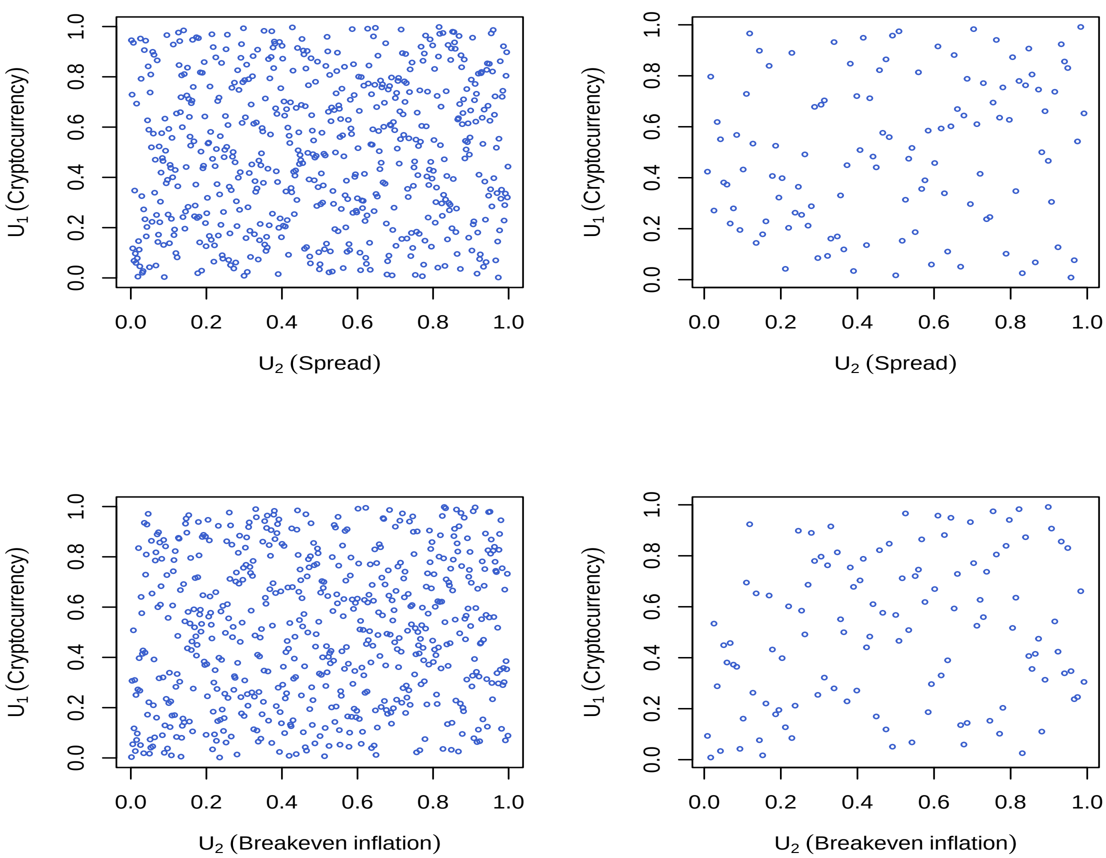
Sticky updating about future uncertainty may play a role in the weak dependence between the CM and the stock market (see e.g., Lochstoer & Muir 2022). This would be a result of investors taking time to incorporate new information regarding future uncertainty from the S&P 500 into their cryptocurrency investment decisions. In this setting, models that can account for the lag in the response of the CM to the volatility forecast of the S&P500 would be more insightful than the copula framework. Moreover, investors may value idiosyncratic risks more than a broad indicator like the VIX. This would, for instance, express in CM participants having a stronger reaction to change in cryptocurrency regulations than broader uncertainty information hidden in the VIX, such as changes in the monetary policy stance or gloomy economic forecasts.

Significant inter-temporal variation of the interconnectedness between the CM and the other markets may also explain the weak variation reported in section 4. In a recent paper, Iyer (2022) reports a deeper connexion between crypto-currencies and the US stock market following the COVID-19 shock. The author finds a pearson correlation of 0.01 and 0.36 for the sub-periods 2017-2019 and 2020-2021 respectively. This implies that long time series can hide or offset recent correlational developments between the CM and other markets. In our analysis, we indirectly control for this issue by using the test for point detection. The point detection (stationarity) test has the advantage of identifying structural breaks that affect the margins and the copula parameter. As reported in section 4, we found no clear evidence of non-stationarity for the four core variables. So, it is unlikely that the weak dependence estimates computed for the log-returns on the CM and the other variables are sample-dependent.

We extend the analysis to account for statistical issues that may affect the strength of the interconnectedness of the CM with other markets. For brevity, we keep the robustness check between the returns on the CM and the bond market variables, meaning the interest rate spread and the breakeven inflation. Possible conundrums with cryptocurrency data entail non-synchronous closing times with other markets analysed by Alexander & Dakos (2020) and noise that might have contaminated the cryptocurrency price series during the early trading days. We deal with the latter problem by sketching the pseudo-observations graph for a sample ranging from 1 January 2020 to 31 March 2022. The starting point of the sub-sample is similar with the one considered in Iyer (2022). Furthermore, we use weekly returns to minimize possible problems with non-synchronous closing times. Weekly aggregation allows us to capture a more substantial bulk of the price variation in presence of non-synchronous distortions.

Figure 7 shows no deviations from our previous conclusions, both for the daily and weekly observations. Summing up, recent periods characterized by a growing public oversight over crypto trading activities does not rule out the weak relationship found in the whole sample between contemporaneous values of cryptocurrency returns and the slope of the yield curve. It is important to note that the period in question also corresponds to the advent of smart money into the cryptocurrency investment sphere [11]. It would be natural to expect economic indicators such as the slope of the yield curve to be correlated with cryptocurrency prices over this period. This deduction stems form the fact that institutional investments ought to follow some technical rules and be aligned with market conditions. Otherwise, rationalizing cryptocurrency investments decisions would be a difficult task. So, there is an imperative obligation to shed light on why cryptocurrency prices seem to be detached from market indicators derived from the state of the world's economy.

Figure 7: Dependence representation between log-returns on the CM, interest rate spread and 5-year breakeven inflation



Notes: This figure presents the pseudo-observations of the log-returns on the CM, the interest rate spread and the breakeven inflation. The sample covers the period 1 January 2020 to 31 March 2022. The pseudo-observations are generated from the standardized residuals of a GARCH(1,1) with student's t innovations. For brevity, we only present the plots, which depict a situation of near independence copula between the variables.

Using our crypto index, we indeed find evidence of a rising positive correlation around 2020.

[11]See Fidelity (2021) for an overview on institutional investors on the CM.

We use the same correlation measure as in Iyer (2022) for comparison purposes. The results of this exercise need to be carefully contextualized for a better understanding of the CM. In level (the first row of Figure 8), the pearson correlation measure is high and characterized by sudden movements between positive and negative values. However, this observation is a consequence of the non-stationarity of the variables. When working with returns, the correlation estimate reduced significantly (bottom panel). Put differently, there is a spurious relationship problem at play between the covariates and the crypto index in level [12]. In terms of the magnitude and the statistical significance of the estimate, there is no major contrast between the two sub-periods. The correlation seems to fluctuate around 0.2 and -0.2 with no pattern. So, there is not a specific pattern for the CM to be more connected with macroeconomic fundamentals following the 2020 sub-period.

Our final robustness check involves estimating a simple ADL(2,2) model to account for the inability of the copula framework in this research to account for dynamic relationship between the variables. The low ADL order specification is chosen for simplicity reasons. Using higher lags will also keep the conclusion laid out below intact. Our crucial goal in this exercise is to test how returns on the CM respond to lags of the interest rate spread and the breakeven inflation. Going forward, these insights will be important priors for more advanced modelling work on cryptocurrency returns. The ADL specification for the returns on the CM and the yield spread is of the form:

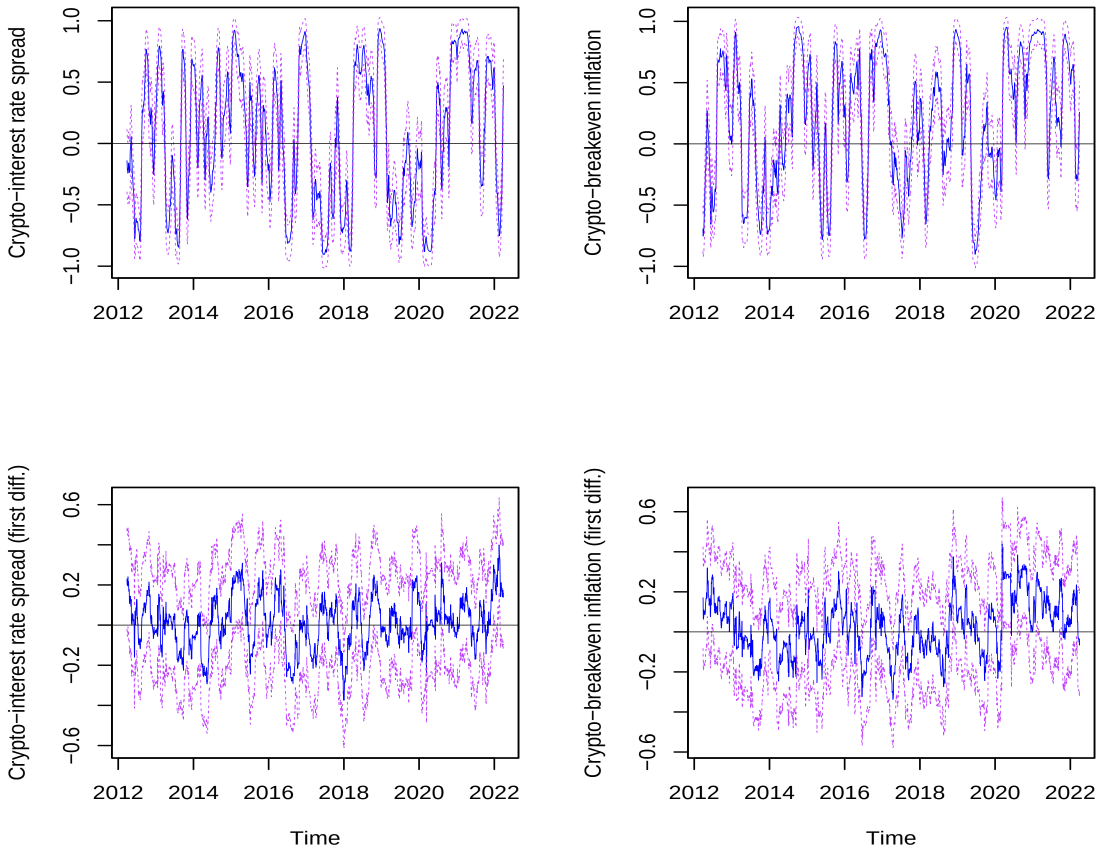
$$X_{1t} = \alpha + \phi_1 X_{1t-1} + \phi_2 X_{1t-2} + \beta_0 X_{2t} + \beta_1 X_{2t-1} + \beta_2 X_{2t-2} + \lambda_0 X_{4t} + \lambda_1 X_{4t-1} + \lambda_2 X_{4t-2} + \epsilon_t. \quad (11)$$

All the variables in the ADL specification are stationary and read as log-returns on the CM, the first difference of the interest rate spread and the first difference of the 5-year US breakeven inflation. We provide the coefficient estimates for the model in Table 9 of the appendix. Both full sample and sub-sample estimates are reported in the table.

The sign of the estimates and their statistical significance carry a number of pivotal lessons to help rationalize the connection between returns on the CM and the wider economic environment. The breakeven inflation affects returns on the CM with one lag when considering a full sample estimation. If today expected inflation increases by 1 percentage point, log-returns on the CM is predicted to increase by 5.6 percentage points. The effect is statistically significant at only 10% level. The coefficients in the first sub-sample period (2012-2019) show no statistical significance at any of the conventional levels. In comparison, we find a strong and significant short-run effect in the second sub-sample (2020-2022). A 1 percentage point increase in the 5-year US breakeven inflation leads to an immediate response of the log-returns on the CM of about 12 percentage points.

[12] We formally test the spurious relationship hypothesis by regressing the crypto index on each of the covariates. The obtained residuals are integrated of order 1. We also reject the Ljung-box null hypothesis of no autocorrelation at 1%.

Figure 8: 60-day rolling correlation between the crypto index, the interest rate spread and the breakeven inflation.



Notes: This figure presents the rolling correlation for the variables. The first row is the variables in level, and the second one reports their variation. The dashed violet lines are the 95% confidence band.

The interest rate spread, or the information contained in it, is not a predictor of price variations on the CM. This result is confirmed by the simple dynamic modelling exercise. The estimates of the spread coefficients are not statistically different from zero in any of the scenari reported in Table 9. The results imply that real returns are important for CM participants. Investors would require higher nominal returns when inflation is expected to go up. More importantly, the coefficients of the ADL model depict a strong contemporaneous effect of the breakeven inflation on returns on the CM in recent times. Overall, including lags is important to uncover dynamic effects of the covariates on the log-returns.

From a practical standpoint, this study helps clear up some misconceptions about potential factors driving cryptocurrency prices. However, the cryptocurrency network remains a fast involving environment with various aspects to understand. The relative absence of entry restrictions makes cryptocurrencies accessible to investors around the world. As such, the CM would probably be one the most diverse investors pool in existence. As a potential downside, this diversity might entail a degree of asymmetry in the financial literacy and the risk attitude of CM market participants. The two factors are likely to affect price variations on the CM. A strand of the behavioral financial literature has for long studied how these distortions feed in trading behaviors (biases) observed from market participants [13]. This facet of the analysis is not covered in this research and would be an important line of investigation for the CM.

5 Conclusion

Our analysis finds no strong dependence between the CM and the US interest rate spread (the breakeven inflation as well) in the copula modelling famework. This result dismisses the importance of the yield curve in investment decision making related to the CM. Such an outcome corresponds to numerically low copula parameters and tail dependence estimates. Our interpretation is supported for the entire sample and a sub-sample analysis. The latter sample covers the post-2020 period, which has seen an expansion of the crypto activities to institutional investors. Our results display the same negligible correlation between the two markets for the sub-sample consideration. In a nutshell, information about future changes in monetary policy and economic cycles contained in the slope of the yield curve does not matter for returns on the CM.

We extend the same methodology to the analysis of the dependence between the volatility on the CM and the VIX. The VIX is known in the literature to be a measure of investors' fear and uncertainty about future market conditions. Our results provide weak evidence that low VIX estimates tend to correspond with high volatility on the CM. This tail dependence relationship becomes stronger when looking at the VIX with a forward-looking volatility estimate for the CM. We interpret this "low-high" volatility result as a shift in resources allocation between the two markets. In times of low volatility, investors, namely the risk-averse ones, would substitute cryptocurrencies for stocks. In the end, the outflow of money would nourish uncertainty and cause the volatility on the CM to spike up. However, the transactions volume in the cryptocurrencies network, approximated by the activity index, does not fully reflect the outpouring interpretation of capital provided for this section (see Figure 6).

However, the conclusion of our analysis changes when we account for the dynamic interactions of the interest rate spread and the breakeven inflation on the log-returns on the CM. Market participants require higher nominal returns on cryptocurrencies when prices are on an increasing trajectory. So, real returns appear to be critical for investors on the CM.

Summing up, our contribution stands in contrast with the strand of the literature that acknowledges a direct significant link between the CM, the monetary policy stance and the government bond market (see e.g., Karau 2021). As such, the result rules out possible risk shifting links that

[13] See Liu et al. (2022) for a brief summary of this strand of the behavioral finance literature.

can explain the emergence of the CM after the 2008 financial crisis. This null result is visible through the weak and non statistically significant result linking the interest rate spread and the log-returns on the CM. On another level, it can be inferred that models that can account for the dynamic between lags of the variables (VAR for instance) would be a robust starting point to address the limitations of the copula modelling. We suggest two methodological points that need to be considered in this regard. First, looking at other ends of the yield curve would be crucial to have a complete picture of the relationship between the CM and the bond market. It is possible that bonds of other maturities might lead to a different conclusion (See e.g., Campbell & Shiller (1991) and the references therein). Second, granular data is an important step to study the profile of CM participants. Shiller (2015) gives an extensive elaboration on "new era thinking" and their connection with unreasonable price valuations. The "new era thinking" refers to the inflated economic optimism that is often fuelled by news outlets. This element of optimism can lead to rising asset prices and, in some cases, to bubble formation. Available granular datasets would be paramount for the investigation of this idea of "new era thinking" in the context of CM possible.

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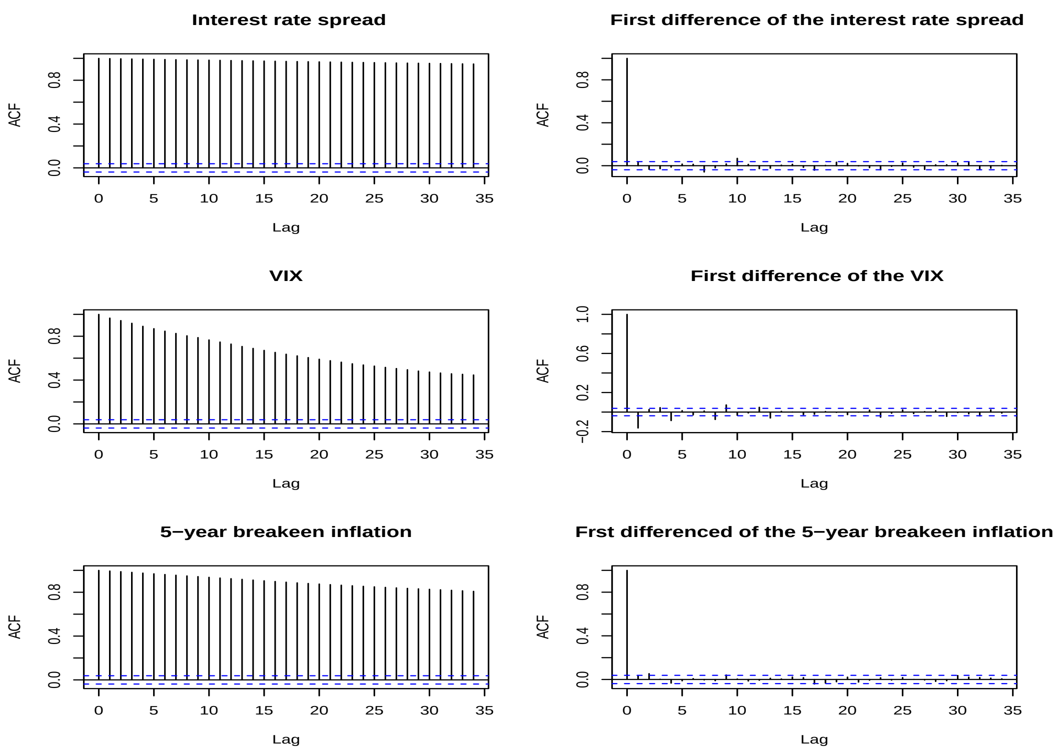
Appendices

A Supplementary information to the summary statistics

Figure 9: ACF of the interest rate spread, the VIX and the breakeven inflation



Notes: This figure presents the ACF and the PACF of the spread series in level. It is clear from the ACF that the series stems from a non-stationary process. Note that the horizontal dashed lines (in red) represent the 95% confidence interval.

B Supplementary information to the modelling section

In choosing the model choice, we start with plots of the correlation between different lags of both series. Figure 10 reveals weak evidence of correlation between successive lag values of X_{1t} and X_{2t}. On the contrary, the squared of both series exhibit significant correlation between contemporaneous and past values. We apply the Ljung-Box test to confirm the existence of serial dependence in the series. There is evidence against serial dependence for X_{1t} and X_{2t} at low lag components (up to 3 for X_{1t} and 6 for X_{2t}). However, the squared transformation of both variables show existence of strong serial dependence up to 12 lags.

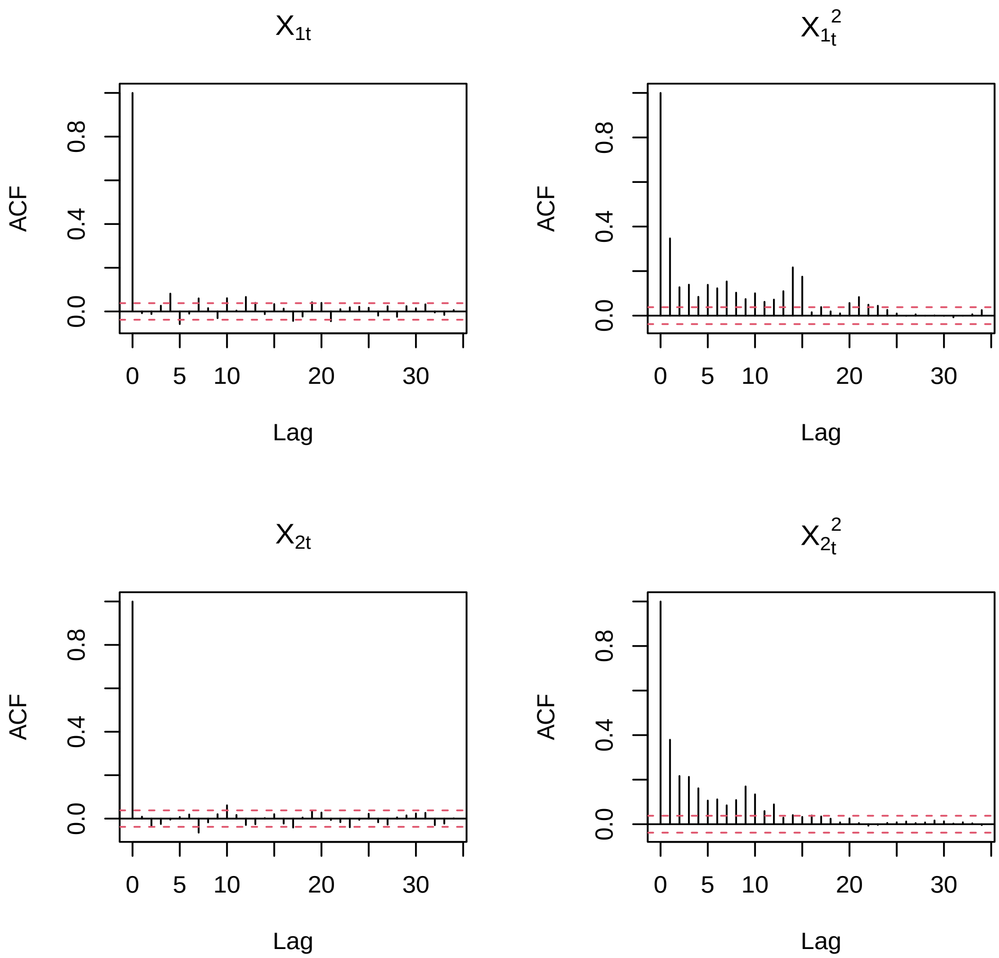
We use stepwise regression to select the number of lags to enter the conditional equations 5 and 6. We construct the variance equation with the squared component of each series. The Bayesian Information Criterion (BIC) selects ARMA(0,0,0)-GARCH(2,1) and ARMA(0,0,0)-ARCH(3) for returns on the CM and change in the spread, respectively.

We use the MLE to compute parameters of the model order suggested by the stepwise regression framework. Estimates are reported in Table 7. In passing, we provide estimates of alternative models for comparison purposes. In the case of X_{1t}, we run a GARCH(1,1) and a TARCH(1,1,1). A GARCH(1,1) sits between the suggested GARCH(2,1) and the TARCH(1,1,1). The GARCH(1,1) represents a simpler framework (less parameters to be estimated), whereas the TARCH (1,1,1) stands as a more complex formulation. The same explanation goes for X_{2t}. Results reported in Table 7 show that the models are equally good according to the AIC. With the BIC, a GARCH(1,1) seems to be a more suitable framework to model the conditional variance of X_{1t} as well as X_{2t}.

To further the model choice analysis, we test for leverage effect. We appraise the asymmetric effect by computing the simple correlation between X_{1t} and its squared component (we follow the same approach for X_{2t}). The correlation is -0.098 and 0.195 for X_{1t} and X_{2t}, respectively. This would suggest a negligible leverage effect on the CM. Unlike the MLE, the TARCH parameter for X_{1t} is not statistically different from zero (at the 10% significance level) when we use the Quasi-Maximum Likelihood Estimation (QMLE) technique. The latter would be a more robust approach to obtain standard errors for estimates of the models in case the normal assumption does not hold for the distribution of residuals. So, analysing the distributional process of the residual series would be an important step to complete the comparison task and decide on the necessity of a TARCH process to model X_{1t}.

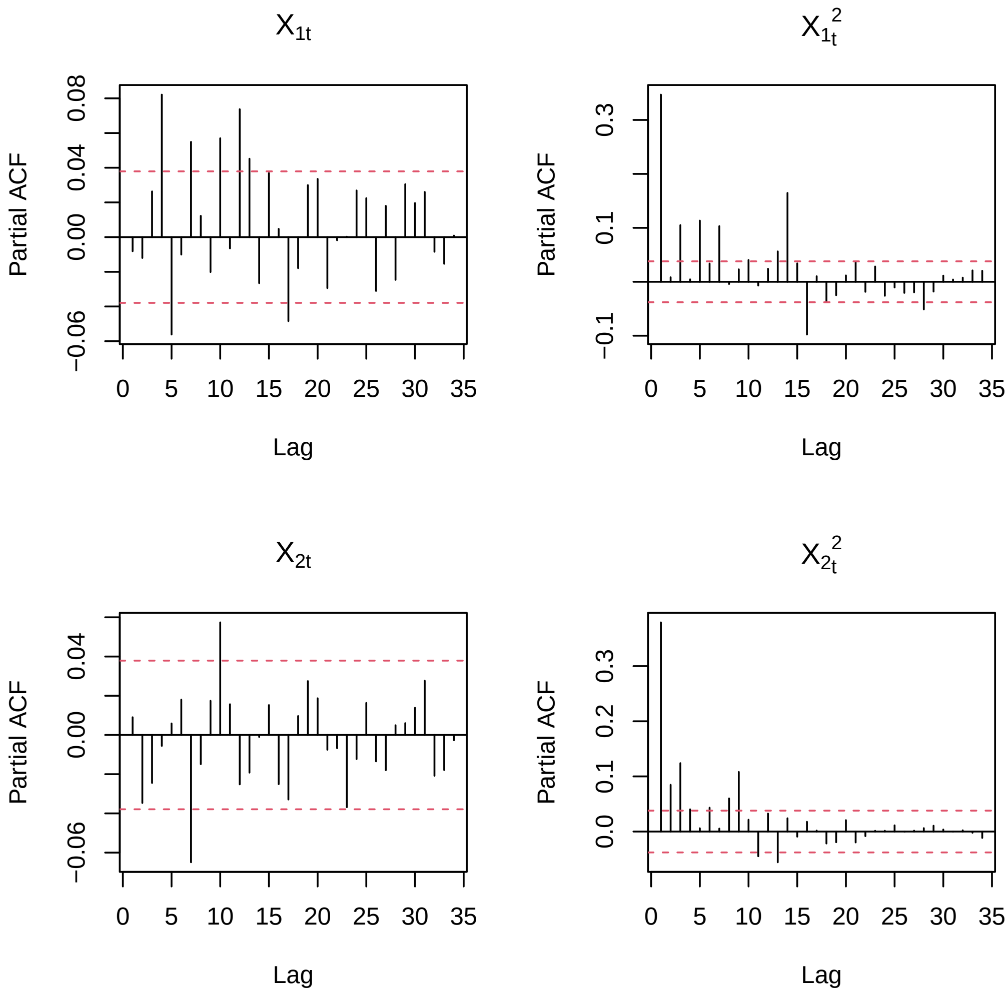
We keep the GARCH(1,1) as the best candidate to model the conditional variance on the CM. On the contrary, the TARCH(1,1,1) would be a good approach for the spread. In fact, a correlation of roughly 0.2 can be relatively informative to appraise asymmetric reactions for the spread series. However, residuals of the TARCH process for X_{2t} are serially correlated. The observation holds for other distributional assumptions on the residuals (see Figure 14). In conclusion, a GARCH(1,1) is considered for both series. We explain the choice of the distributional assumption in Figure 12 and Figure 13. Considerations on the fitness of the models are displayed in Figure 15.

Figure 10: Autocorrelation function of log-returns and change in the spread



Notes: This figure presents the ACF of log-returns on the CM (X_{1t}) and the change in the spread (X_{2t}) . The horizontal dashed lines (in red) represent the 95% confidence interval of a white noise series.

Figure 11: Partial autocorrelation of log-returns and change in the yield



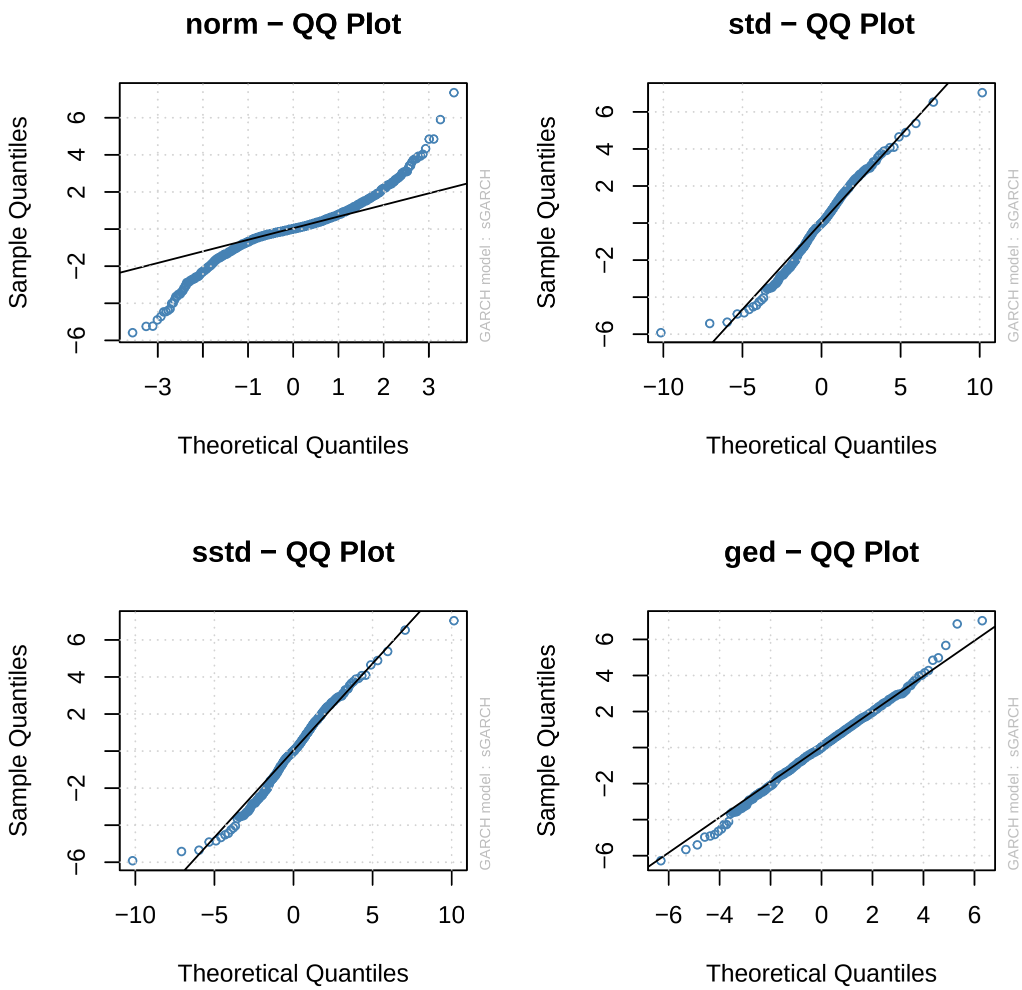
Notes: This figure presents the PACF of log-returns on the CM (X_{1t}) and the change in the spread (X_{2t}) . The horizontal dashed lines (in red) represent the 95% confidence interval of a white noise series.

Table 7: Model comparison for returns on the CM and change in spread

	X_{1t}			X_{2t}		
	GARCH(2,1)	GARCH(1,1)	TARCH(1,1,1)	ARCH(3)	GARCH(1,1)	TARCH(1,1,1)
ω	0.0001***	0.0001***	0.002***	0.000***	0.000	0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
α_1	0.120***	0.139***	0.155***	0.142***	0.073***	0.065***
	(0.022)	(0.014)	(0.022)	(0.023)	(0.016)	(0.011)
α_2	0.027			0.048***		
	(0.027)			(0.017)		
α_3				0.081***		
				(0.019)		
β_1	0.829***	0.839***	0.847***		0.901***	0.904***
	(0.019)	(0.015)	(0.013)		(0.014)	(0.021)
β_2						
γ_1			−0.093**			−0.180*
			(0.041)			(0.101)
AIC	−3.452	−3.452	−3.452	−12.660	−12.670	−12.670
BIC	−3.441	−3.443	−3.441	−12.649	−12.662	−12.659

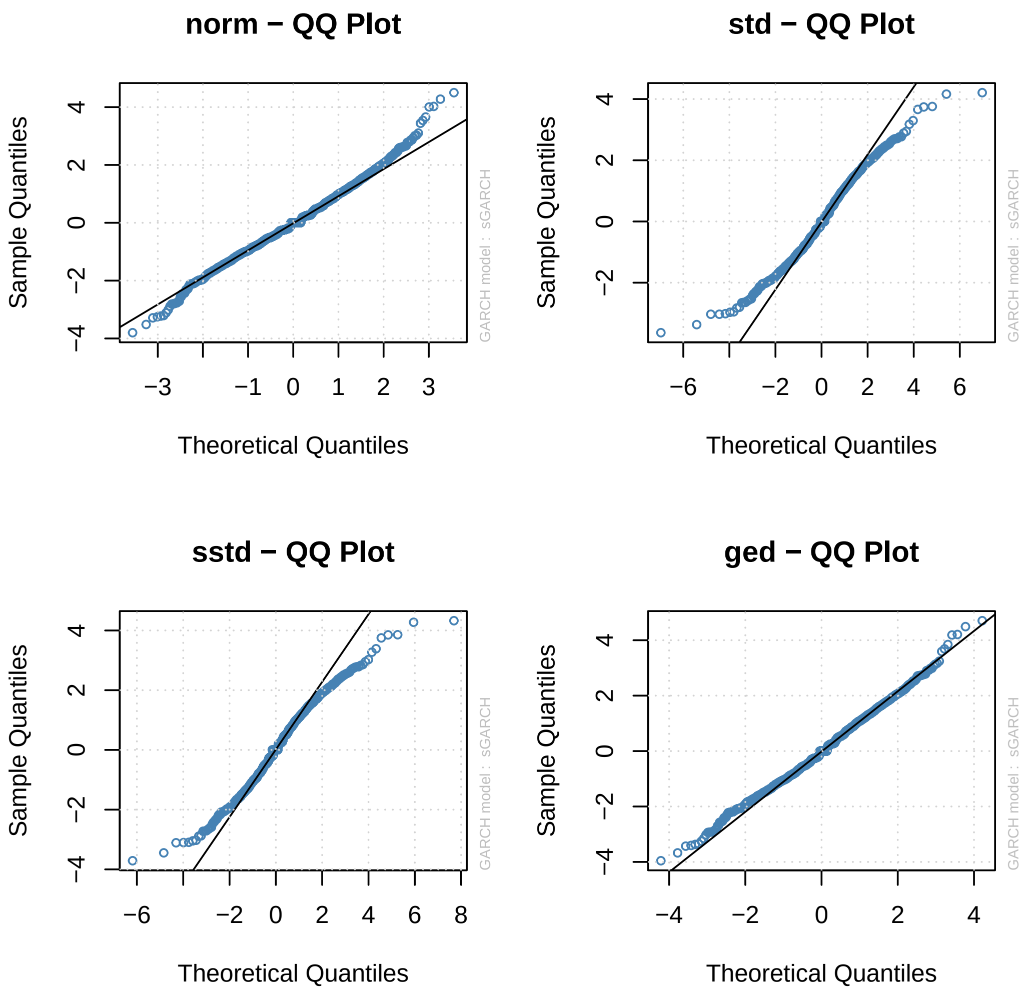
Notes: This table presents different model candidates for the conditional variance of X_{1t} and X_{2t}. All models assume normal innovations. The standard errors with the QMLE is 0.087 and 0.107 for the TARCH parameter in the X_{1t} and X_{2t} cases respectively.
***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.

Figure 12: Comparison of different distribution assumption for X_{1t} innovations



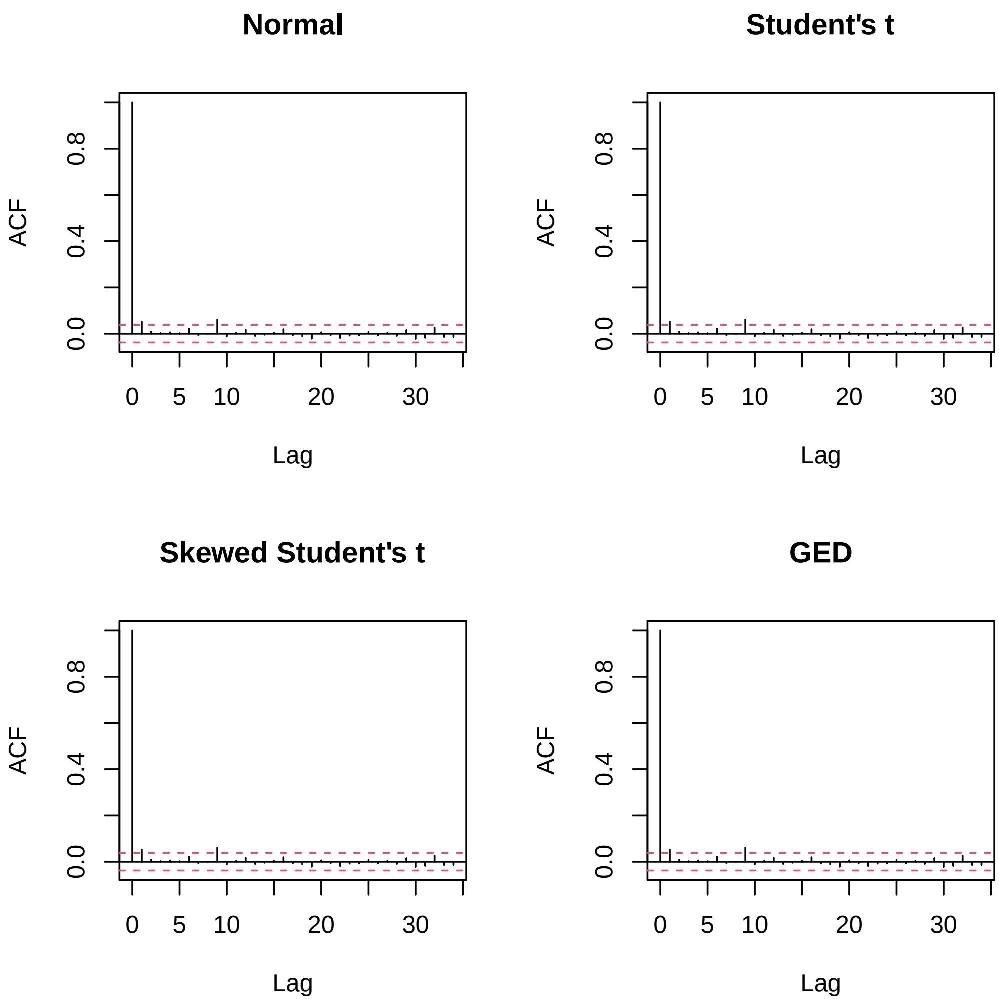
Notes: This figure plots the residuals of the ARMA(0,0,0)-GARCH(1,1) process against the normal, the student's t, the skewed student's t, and the Generalized Error Distribution (GED).The normal assumption offers the worst fit out of all the assumptions considered. The remainder of the models seems to be adequate to account for the residuals variation. However, the shape parameter estimate is less than one (0.852) with the GED, which rules out this assumption as a suitable option.

Figure 13: Comparison of different distribution assumption for X_{2t} innovations



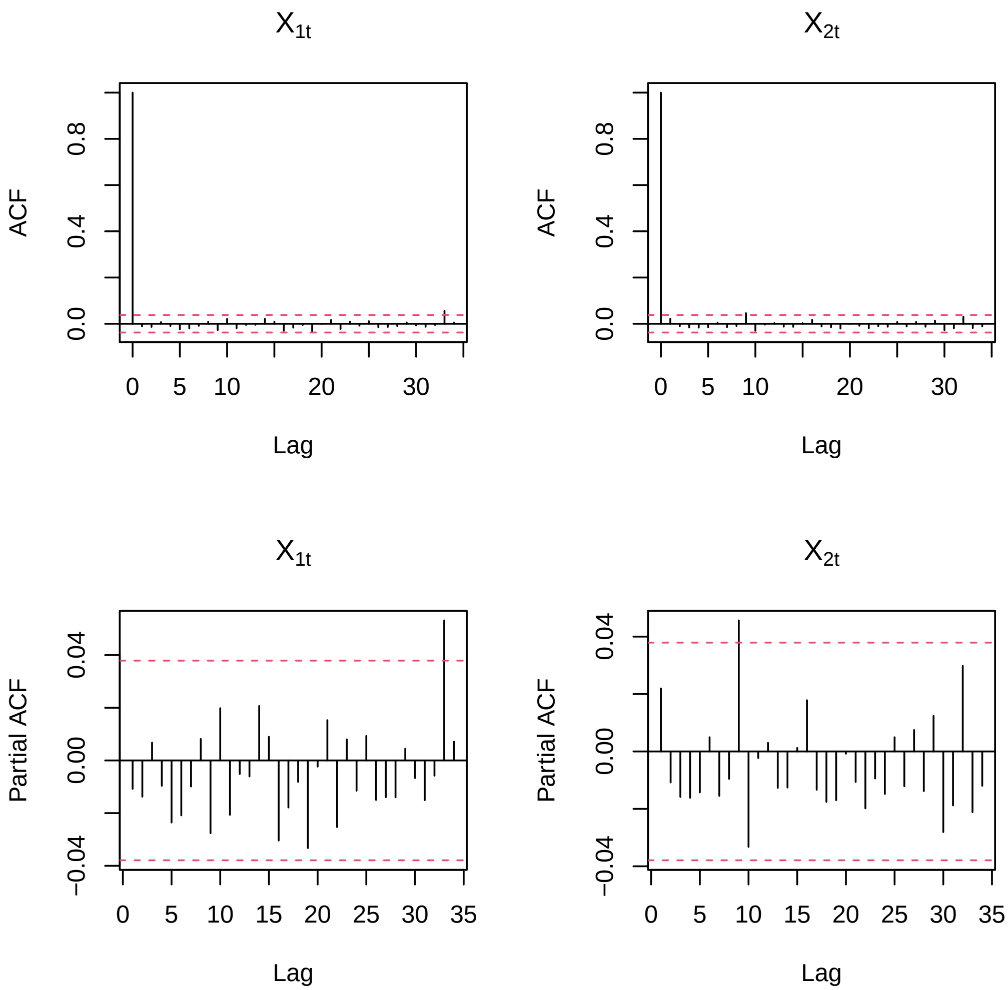
Notes: This figure plots the residuals of the ARMA(0,0,0)-GARCH(1,1) process against the normal, the student's t, the skewed student's t, and the GED. Normal and GED look appropriate to explain the dynamics of the residuals, with some trivial difference between the two distributions. The likelihood ratio test gives evidence that a GARCH(1,1) with normal residuals is optimal for the problem at hand.

Figure 14: Squared residuals from the TARCH(1,1,1) of the X_{2t} series



Notes: This figure plots the ACF of squared residuals for the change in the spread series. It can be inferred from the graphs that there is significant autocorrelation at lag 1. The Ljung-box test supports the evidence of serial correlation at 1% significance level.

Figure 15: Squared residuals of X_{1t} and X_{2t} with GED innovations



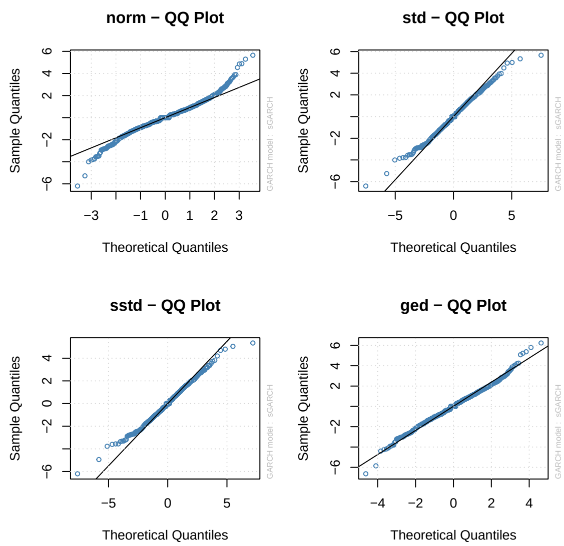
Notes: This figure plots the ACF and PACF of squared residuals for both variables. The processes exhibit a white noise behavior. We then formally test for serial dependence with the Ljung-Box (up to lag 12). We found no evidence of serial dependence for X_{1t} (p-value=0.753) and X_{2t} (p-value=0.395). So, the GARCH processes we choose are adequate to model the conditional variance of X_{1t} and X_{2t}.

Table 8: CM and VIX copula estimates

	Normal	Student's t	Clayton	Frank	Gumbel-Hougaard	Joe
θ	0.105***	0.105***	0.109***	0.488**	1.062***	1.099***
	(0.020)	(0.020)	(0.029)	(0.188)	(0.015)	(0.023)
τ^L		0.000	0.002***			
		(1×10^{-8})	(5.016×10^{-5})			
τ^U		0.000			0.08***	0.121***
		(1×10^{-8})			(0.001)	(0.003)
Deg. of freedom		9462				
Log. Likelihood	14.48	14.48	-	8.71	14.23	16.29
AIC	26.968	20.961	-	15.434	24.466	28.589

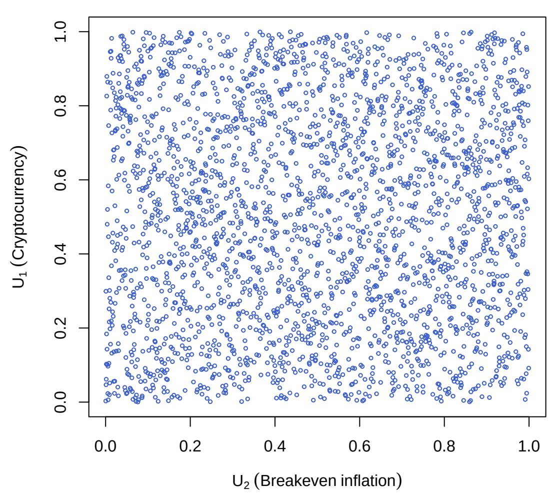
Notes: This table presents estimates of some of the widely used copulas. In parentheses are standard errors of the estimates. In the spirit of Table 2, the statistical significance of the Joe and Gumbel-Hungaard is tested as $H_0 : \theta = 1$ and $H_1 : \theta > 1$. The standard errors for the tail probabilities are computed with the delta method. The MLE failed to estimate the dependence parameter in the Clayton case. We instead estimate the parameter via the method-of-moment (Spearman's rho). Hofert et al. (2019) gives a detailed explanation of this technique.
***Significant at the 1 percent level.
***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level

Figure 16: Residuals of the breakeven GARCH(1,1) model



Notes: This set of plots presents the 4 assumptions considered to model the volatility of the breakeven inflation. The GED shows a better fit with respect to the other distributions. The intercept of the conditional variance equation is approximately zero (same for the mean equation). The conditional variance equation can be written as $\delta_{4t}^2 = 0.058\epsilon_{4t-1}^2 + 0.901\delta_{4t-1}^2$. The GED shape parameter is estimated to be 0.9.

Figure 17: Pseudo-observation of log-returns on the CM and the first difference of the inflation expectation



Notes: This figure presents the pseudo-observations of the residuals (uniform transformation) from the GARCH equations for the log-returns on the CM and the 5-year breakeven inflation (the differenced series). The spread of the points shows no particular dependence structure.

Table 9: Full sample and Sub-sample estimates of the ADL model

	X_{1t}	X_{1t}	X_{1t}
	Full sample (2012-2022)	Sub-sample (2012-2019)	Sub-sample (2020-2022)
α	0.0034***	0.004***	0.003
	(0.001)	(0.001)	(0.002)
ϕ_1	−0.010	−0.008	−0.023
	(0.019)	(0.021)	(0.042)
ϕ_2	−0.015	−0.028	0.049
	(0.019)	(0.022)	(0.042)
β_0	0.921	1.024	2.941
	(2.402)	(2.977)	(3.839)
β_1	−1.03	−0.503	−1.642
	(2.403)	(2.978)	(3.848)
β_2	−1.513	−2.029	0.836
	(2.405)	(2.975)	(3.813)
λ_0	2.758	−2.903	11.758***
	(2.927)	(3.869)	(4.106)
λ_1	5.570*	4.2	5.6
	(2.927)	(3.876)	(4.238)
λ_2	2.414	2.114	1.2
	(2.927)	(3.868)	(4.148)
N. of obs.	**2673**	**2086**	**587**

Notes: This table presents a simple ADL model to capture the dynamic between returns on the CM, the interest rate spread and the breakeven inflation. 1 January 2020 is the cut-off point for the subsamples as in Iyer (2022). We find no evidence to reject the hypothesis that the residuals of the above equations are white noise up to lag 3. The p-values from the ljung-box test are 0.623, 0.556 and 0.993 for the full sample, the first sub-sample (2012-2019) and the second subsample (2020-2022) respectively.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

*Significant at the 10 percent level