Currency Competition and Monetary Non-neutrality

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Abstract

We build a theoretical model where both fiat money and crypto-currencies are used as media of exchange for differentiated goods. Crypto-currencies offer pecuniary benefits, such as avoiding consumption taxes, and non-pecuniary benefits like transaction privacy, while non-users face utility losses that grow with available goods. We identify an endogenous threshold good where consumers are indifferent between government-backed money and privately-issued currency, leading to three equilibrium scenarios: all goods purchased with fiat, all with crypto, or a mix of both. Our model predicts that, while fiat money is neutral, crypto-currencies are non-neutral due to mining costs, which affect labor allocation.

Keywords : Crypto-currency, fiat money, currency substitution, neutrality and non-neutrality.

1 Introduction

Crypto-currencies emerged with the promise of eliminating frictions that lead to high transaction costs in traditional money-based economies (see Nakamoto (2008)). This alternative medium of payment presents consumers with a decision-making problem regarding which currency to use for day-to-day transactions—whether to choose fiat money or crypto-currency. In a two-country model without capital controls, the choice of currency would depend on the real exchange rate between the two. However, crypto-currencies offer more than just liquidity services. For instance, anonymity in transactions is a non-pecuniary benefit linked to crypto-currencies, adding complexity to this decision. Furthermore, the challenge lies in how crypto-currencies can coexist with fiat money. This raises important questions: what factors determine the stock of goods purchased with either form of currency? And what are the welfare-enhancing attributes associated with higher adoption of crypto-currencies?

Current theories remain inadequate in formalizing an explanation of the determinants of the demand for *crypto-purchased goods* and *money-purchased goods*. For instance, Marchiori (2021) addresses this problem in a cash-in-advance model where both crypto-currencies and fiat money

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serve as means of payment for consumption goods. However, the model imposes a strong exogenous constraint on a fixed set of goods that must be purchased with either crypto-currencies or fiat money. Benigno et al. (2022) construct a similar theoretical framework but place analytical emphasis on the monetary policy implications of an economy with the two currencies available for transactions. To that end, both models offer insufficient insights on the questions raised in the paragraph above.

We propose a one-period cash-in-advance model in which both fiat money and privately issued currency are accepted as media of exchange for a continuum of differentiated goods. However, an exogenous rule requires that all production factors be remunerated in money terms. We assume heterogeneity in access to crypto-currencies, reflecting differences in information and technology (IT) know-how between crypto users and non-users. In subsection 5.4, we show that an increase in the crypto-currency accessibility parameter has real economic consequences, particularly in terms of consumption and labor reallocation across sectors of the economy

A key element in our analysis is the modeling of crypto-currencies as a tool that confers both *pecuniary* and *non-pecuniary* benefits to consumers. In the former case, consumers can use crypto-currencies to bypass value-added taxes on consumption goods. In the latter case, crypto-currencies impose a utility loss on non-users in the form of a lack of privacy in the goods market. This loss is a linear and increasing function of the set of goods available to consumers, meaning that as more goods become available, the loss grows proportionally. Building on these features, we argue in subsection 3.2 for the existence of a *threshold good*, where consumers are indifferent between either fiat money or crypto-currency, thus giving an explicit payment role to the latter in the goods market.

Our first set of results lead to three possible outcomes based on the relative transaction costs of purchased goods. In the first scenario, all goods are purchased with money due to high crypto-fees or low transaction costs for money. In the second scenario, low crypto-fees or high taxes on money lead to the exclusive use of crypto-currencies for transactions. The third scenario introduces a threshold, where both currencies are used, with the set of goods purchased with money expanding as crypto-fees rise or consumption taxes on money payments fall.

In our model, fiat money is costlessly produced and distributed to final goods consumers. We exclude savings in the form of money holdings or capital investments. This leads to an equilibrium solution where money is neutral in the economy, which is a standard result in the real business cycle literature. On the other hand, mining crypto-currencies is costly in the model. As a result, crypto-currencies exhibit non-neutrality in the system because the presence of crypto-currency mining activity affects labor allocation across sectors. This effect is visible in the numerical simulation below, where we show that a positive investment shock in the mining sector influences real wages and drives up unemployment in the goods production sector.

The remainder of the paper progresses as follows. We first present the connection between our results and the existing literature in section 2. We proceed to model the different components of our general equilibrium framework in section 3. Then, we explore the equilibrium conditions emerging from the optimization problem in section 4. Finally, we provide a numerical analysis to study the response of the endogenous variables following a shock to the exogenous constants in the final section.

2 Related Literature

The analysis in this paper advances the growing literature on the co-existence of government-backed and privately-issued currencies. Work on currency competition and the concept of private currencies is well established in the monetary literature (see Hayek (1976) and Kareken & Wallace (1981)). Benigno et al. (2022) provide an extensive discussion of currency competition and its various ramifications with the crypto-currency framework. For the sake of clarity, we focus on contributions related to crypto-currencies and highlight the place of our analysis within this body of work.

A closely related analytical setup to ours is the analysis by Schilling & Uhlig (2019b) on the medium of exchange role of fiat money and crypto-currencies. Their model predicts the existence of an endogenous good for which consumers are indifferent between fiat money and crypto-currencies. Other elements, such as the difference in transaction costs for crypto-using and money-using consumers, are also present in our paper. A key point of departure between the two frameworks lies in our treatment of crypto-currencies as an instrument that facilitates privacy in goods transactions, which is explicitly modeled in our framework. Other contributions in this line of research abstract from the privacy aspect and present further modeling divergences from our analysis. For instance, Marchiori (2021) restricts the transactive role of crypto-currencies to a specific set of goods. In Fernández-Villaverde & Sanches (2019), the existence of crypto-currencies can lead to an undesirable equilibrium where the stock of money in circulation fails to meet transaction needs. Additionally, Yu (2023) and Schilling & Uhlig (2019a) derive a role for crypto-currencies when their rate of return matches that of fiat money in the market. Another currency competition model is proposed by Zhu & Hendry (2019), which focuses on price stability. Our paper, however, does not incorporate most of the features discussed in these works. Instead, we focus on deriving the conditions that make currency substitution possible in an economy with both private currency and fiat money.

3 Static Model with Purely Transactive Currencies

3.1 Basics

Consider a static economy where consumption transactions can be performed using two different currencies. One currency, called *money*, acts as the legal tender and is issued by the government. The other currency is represented by privately-issued virtual coins and is called *crypto-currency*. Households supply labor and physical capital to firms and receive factor incomes that, as a result of exogenous rules, must be paid with *money*. The hypothesis that all production factors must be remunerated using the legal tender of the economy essentially plays the role of a cash-in-advance

(CIA) constraint in a one-period economy. However, the goods market is open to alternative means of payments, and the crypto-currency can be used to purchase consumption goods. Households can use part of their money stock to purchase the crypto-currency on *exchange platforms* that charge fees for their services. At the same time, firms receiving payments in crypto-currency will use the exchange platform to convert crypto-payments into money that will be used to remunerate labor and capital owners. Like consumers, firms will be charged a *crypto-transaction fee* since exchange platforms bear the costs of validating every transaction that involves crypto-currency.

The static environment has specific characteristics. Since households spend all their income in one period and the crypto-currency is only used for transaction purposes, the stock of crypto-currency returns in full to the exchange platform when all exchanges are completed. The final state of the crypto-market would obviously be radically different in a dynamic model including multiple periods and saving-investment opportunities. Nonetheless, this static model offers important insights on how the introduction of a crypto-currency may affect the equilibrium in CIA-constrained economies, as we will see in the dynamic model of the next section.

3.2 Consumers

The economy is populated by L consumers, each consuming (a continuous finite mass of) N different goods indexed by $n \in [0, N]$. Each good n may in principle be purchased using money or crypto-currency. However, only a fraction ϵ of the L consumers has access to crypto-currencies and can freely decide which goods to buy with either means of payment. The remaining $(1-\epsilon)\,L$ consumers purchase all the N goods using money. We treat $\epsilon \in (0, 1)$ as an exogenous parameter that we can manipulate to investigate important properties of the model. Letting $\epsilon \to 0$ we can study the equilibrium of a benchmark economy without crypto-currency and compare its predictions to those obtained in the general case, $0 < \epsilon < 1$, as well as in the opposite polar case where every household has access to crypto-currency, $\epsilon \to 1$. It is safe to argue that $0 < \epsilon < 1$ is a realistic hypothesis – e.g., because crypto-currencies are not used by households with insufficient IT literacy or equipment. However, the key rationale for our hypothesis $\epsilon < 1$ is that it provides us with a free parameter whereby we can assess the impact of market-size shocks – that is, exogenous changes in the potential demand for cryptocurrencies – and more generally the welfare effects of crypto-currencies.

In order to distinguish individual variables that refer to either type of households, we will use 'tildas': for any variable x associated with consumers having access to the crypto-currency, the same variable for 'crypto-less consumers' will be denoted by $\tilde{x}$. The next two sub-sections specify the expenditure problem for each type of consumer in turn.

3.2.1 The crypto-less consumer

Consider an individual within the set of $(1-\epsilon)L$ consumers having *no access* to the crypto-currency. Total utility from consumption is an integral of well-behaved sub-utility functions,

$$\tilde{U} \equiv \int_0^N \tilde{u}_n(\tilde{c}(n))\, dn \tag{1}$$

where $\tilde{c}(n)$ is the consumed quantity of the n-th good.

The crypto-less consumer purchases all goods using money, which entails different types of transaction costs. The literature suggests a long list of private costs associated with money payments that could be circumvented using alternatives like crypto-currencies. Some costs are *non-pecuniary* – e.g., lack of anonymity, legal constraints, personal time costs generated by bureaucracy, red-tape, transaction-recording and similar administrative duties – and are typically connected to the nature of the good being purchased regardless of its market value: the transaction per se creates disutility and can thus be modeled as a non-distortionary 'tax' in terms of utility. Other costs are *pecuniary* – e.g., credit-card and money-transfer fees, intermediation costs, consumption taxes imposed by governments on traceable money transactions – and can be either lump-sum or distortionary. To cover all bases, our model includes both pecuniary and non-pecuniary costs.

We capture non-pecuniary costs by introducing a simple disutility term: if good n is purchased with money, the associated net satisfaction is

$$\tilde{u}_n(\tilde{c}(n)) = \ln[\tilde{c}(n)\cdot(1-\delta(n))] \text{ with } 0 \leqslant \delta(n) < 1. \tag{2}$$

The disutility parameter $\delta(n)$ is good-specific because non-pecuniary transaction costs may vary substantially across types of goods. The logarithmic form (2) rules out distortions in the sense that, under utility-maximizing conditions, goods characterized by different disutility parameters $\delta(\cdot)$ will capture identical expenditure shares. In other words, non-pecuniary transaction costs are a non-distortionary tax in terms of utility.

The pecuniary costs of money-purchased goods, instead, are represented by a proportional fee: consumers buying good n using *money* will spend

$$p(n)\cdot\tilde{c}(n)\cdot(1+\tau),$$

where $p(n)$ is the market price of the good in terms of money, and $\tau > 0$ is the fee rate. The money-transaction fee τ may be interpreted in several ways – e.g., as a credit-card fee charged by private intermediaries, a consumption tax set by the government, a sunk monetary cost not collected by other agents. For our purposes, the key characteristic is that the money-transaction fee will not apply if the same good is purchased using the crypto-currency. In this model, τ is a consumption tax rate that the government is able to impose exclusively on recorded money payments for consumption. This will imply a tax-avoidance benefit from using crypto-currency.[1]. Alternative models in which τ is a transaction fee paid to banks will likely yield similar results as

[1] This property of the model is empirically plausible: there is widespread consensus that the use of crypto-

long as the same fee does not apply to crypto-payments. The expenditure problem of the crypto-less consumer is

$$\max_{\{\tilde{c}(n)\}} \tilde{U} = \int_0^N \ln\left[\tilde{c}(n) \cdot (1 - \delta(n))\right] dn$$

subject to

$$\tilde{x} = \int_0^N p(n)\, \tilde{c}(n) \cdot (1 + \tau)\, dn \tag{3}$$

where $\tilde{x}$ is individual spending on consumption goods. The first order conditions imply identical expenditure shares for each good,

$$p(n)\, \tilde{c}(n) \cdot (1 + \tau) = \tilde{x}/N \quad \text{for each } n \in [0, N]. \tag{4}$$

As previously noted, non-pecuniary costs do not distort expenditure shares: crypto-less consumers simply suffer a deadweight utility loss by purchasing goods using money since they do not have access to the crypto-currency. The individual income constraint of crypto-less consumers reads

$$\tilde{x} = w + rk_i + \tilde{g} \tag{5}$$

were w is the prevailing wage rate, r is the rental rate of individually-owned capital k_i, and $\tilde{g}$ represents lump-sum transfers from the government. The implicit hypotheses of (5) are that each individual supplies one unit of homogeneous labor that is remunerated at the same rate w by all firms and sectors, and rents k_i units of capital to goods-producing firms obtaining the same rental rate r. The goverment uses transfers $\tilde{g}$ to rebate the proceeds from the consumption tax to households.

3.2.2 The crypto-user: preferences

Consider an individual within the set of ϵL consumers having access to the crypto-currency. In the present environment, the justification for the existence of the crypto-currency is that using money to purchase at least some types of goods entails private transaction costs that exceed those implied by using the crypto-currency. In general, the utility of crypto-using consumers can be written as

$$U \equiv \int_0^{\bar{n}} u_j(c(j))\, dj + \int_{\bar{n}}^N u_i(c(i))\, di \tag{6}$$

where $c(n)$ is the physical quantity of the n-th good consumed, for any $n \in [0, N]$. The right hand side of (6) distinguishes between subsets of goods purchased using different means of payment:

currencies can facilitate tax-avoidance as well as similar shadow-economy activities. The latter point is explained and documented in a recent publication of the Bank for International Settlements (BIS), see the BIS (2023) report for details. Other narratives on the use of crypto-currencies includes a greater penetration in places with tight capital control (see Makarov & Schoar (2019)).

the subset indexed by $j \in [0, \bar{n})$ is purchased using *money* and the associated utility u_j includes non-pecuniary costs just like expression (2) above,

$$u_j\left(c\left(j\right)\right) = \ln\left[c\left(j\right) \cdot \left(1 - \delta\left(j\right)\right)\right] \text{ for } j \in [0, \bar{n}), \tag{7}$$

whereas the subset of goods indexed by $i \in [\bar{n}, N]$ is purchased using the *crypto-currency* and the associated utility does not include the disutility term:

$$u_i\left(c\left(i\right)\right) = \ln c\left(i\right) \text{ for } i \in [\bar{n}, N]. \tag{8}$$

Expressions (6)-(8) implicitly define a *threshold good*, indexed by $n = \bar{n}$, which splits the set $[0, N]$ by payment characteristics. In related literature, different means of payments are associated to different goods in a pre-determined way. For example, in the Marchiori (2021) model there exists one 'cash good' that can only be purchased with money and one 'virtual good' that can only be purchased with crypto-currency by assumption. In our analysis, instead, all the N goods can in principle be purchased using either type of currency, but consumers choose which goods to pay with either method according to utility maximization, so that the *threshold good* $\bar{n}$ is endogenously determined by preferences and market conditions. Therefore, our model justifies the existence of the crypto-currency for transactive purposes, and will predict that changing market conditions, or exogenous shocks on relevant parameters, will affect the transactive demand for crypto-currency even along the extensive margin via changes in the endogenous threshold $\bar{n}$.

The existence of a threshold good depends on the distribution of disutility terms across goods. We model such distribution in the simplest way by specifying $\delta\left(n\right)$ as a function that, under very mild assumptions, determines an interior cut-off point $\bar{n} \in (0, N)$ whereby both the resulting subsets, $[0, \bar{n})$ and $[\bar{n}, N]$, are non-empty. This result (i.e., the existence of subsets of goods purchased with different means of payments) hinges on the existence of good-specific costs associated with money payments.[2] In the next two subsections, we solve the expenditure problem of the crypto-user and then determine the threshold good by specifying a suitable function $\delta\left(n\right)$.

3.2.3 The crypto-user: expenditure problem

We solve the consumer problem in two steps. We firstly derive the utility-maximizing conditions taking $\bar{n}$ as given. Secondly, we derive the no-arbitrage condition for the threshold good $\bar{n}$ taking expenditure shares as given. The key elements are the opportunity costs of alternative means of payments. Purchasing $c\left(n\right)$ units of good n using money requires, as we know, spending

$$p\left(n\right) \cdot c\left(n\right) \cdot \left(1 + \tau\right). \tag{9}$$

[2] Besides all the possible interpretations of δ and τ, what matters for our analysis is that at least one type of money transaction costs – pecuniary or non-pecuniary – is good-specific. Our results are essentially the same if we exclude pecuniary transaction costs ($\tau = 0$) while keeping $\delta\left(n\right)$ good-specific, or vice versa, if we exclude disutility costs ($\delta = 0$) and assume, instead, good-specific pecuniary costs, $\tau\left(n\right)$. Modelling the general case where both $\delta\left(n\right)$ and $\tau\left(n\right)$ are good-specific creates unnecessary algebraic complications without yielding further economic insight.

Purchasing the same good using crypto-currency requires the consumer to convert the necessary amount of money holdings into crypto-currency and purchase the good from the producer by transferring the crypto-currency to the latter. The exchange platform will charge a fee for the currency exchange service. Crypto-transaction fees are proportional to the amount of crypto-currency involved: exchange platforms apply the rate φ^C to consumers selling money against crypto. The total cost to the consumer, expressed in terms of money, is

$$Q \cdot p^* (n) \cdot c(n) \cdot \left(1 + \varphi^C\right), \tag{10}$$

where $p^* (n)$ is the price of the n-th good expressed in crypto-currency units, Q is the *nominal exchange rate* – i.e., the units of money needed to purchase one unit of crypto-currency besides the fee rate φ^C that the exchange platform charges on consumers selling money against crypto.

Expenditure problem for given $\bar{n}$. Given the existence of a unique interior cut-off point $\bar{n} \in (0, N)$, the expenditure problem solved by the consumer is

$$\max_{\{c(j),c(i)\}} \int_0^{\bar{n}} \ln \left[c(j)(1 - \delta(j))\right] dj + \int_{\bar{n}}^N \ln c(i)\, di$$

subject to

$$x = \int_0^{\bar{n}} p(j)\, c(j)(1 + \tau)\, dj + \int_{\bar{n}}^N Q p^* (i)\, c(i) \left(1 + \varphi^C\right) di, \tag{11}$$

where x is consumption expenditure per capita in monetary terms. Denoting by λ the multiplier for constraint (11), the first order conditions read

$$1 = \lambda p(j)\, c(j)(1 + \tau) \quad \text{for each } j \in [0, \bar{n}), \tag{12}$$

$$1 = \lambda Q p^* (i)\, c(i) \left(1 + \varphi^C\right) \quad \text{for each } i \in [\bar{n}, N]. \tag{13}$$

Combining these expressions to eliminate λ, we obtain identical expenditure levels for each good,

$$Q p^* (i)\, c(i) \left(1 + \varphi^C\right) = \frac{x}{N} \text{ and } p(j)\, c(j)(1 + \tau) = \frac{x}{N}. \tag{14}$$

Aggregation of goods by type of payment yields

$$\int_0^{\bar{n}} p(j)\, c(j)(1 + \tau)\, .dj = \bar{n} \cdot p(j)\, c(j)(1 + \tau) = \frac{\bar{n}}{N} \cdot x, \tag{15}$$

$$\int_{\bar{n}}^N Q p^* (i)\, c(i) \left(1 + \varphi^C\right) di = (N - \bar{n}) \cdot Q p^* (i)\, c(i) \left(1 + \varphi^C\right) = \frac{N - \bar{n}}{N} \cdot x. \tag{16}$$

The individual income constraint of crypto-using consumers reads

$$x = w + r k_i + g \tag{17}$$

and has the same interpretation as (5). For future reference, note that result (14) implies

$$\frac{c(j)}{c(i)} = \frac{Q p^* (i)}{p(j)} \cdot \frac{1 + \varphi^C}{1 + \tau}. \tag{18}$$

Expression (18) is the relative demand of crypto-purchased versus money-purchased goods.

Conditions for using money versus crypto-currency. In order to choose the best payment option for a given good $n \in [0, N]$, the consumer compares opportunity costs in terms of utility. We can think of this choice as a sub-problem in which the consumer compares the utility levels enjoyed by spending a fixed amount of income $\hat{x}$ on good n using alternative payment methods, and then chooses the method yielding the highest utility. From (9) and (10), the hypothetical consumption levels attained under money- and crypto-payments are

$$\tilde{c}' = \frac{\hat{x}}{p(n) \cdot (1+\tau)} \quad \text{and} \quad \tilde{c}'' = \frac{\hat{x}}{Qp^*(n)\, c(n)\, (1+\varphi^C)}, \tag{19}$$

where $\tilde{c}'$ is purchased using money and $\tilde{c}''$ is purchased using the crypto-currency. Calculating the associated utility levels $u_n(\tilde{c}')$ and $u_n(\tilde{c}'')$ from (7)-(7), the welfare gap reads

$$u_n\left(\tilde{c}'\right) - u_n\left(\tilde{c}''\right) = \ln\left[\frac{Qp^*(n)}{p(n)} \cdot \frac{\left(1+\varphi^C\right)(1-\delta(n))}{(1+\tau)}\right]. \tag{20}$$

Therefore, the condition for using the crypto-currency, $u_n(\tilde{c}') \leqslant u_n(\tilde{c}'')$, is

$$Qp^*(n) \cdot \left(1+\varphi^C\right) \leqslant p(n) \cdot \frac{1+\tau}{1-\delta(n)}. \tag{21}$$

Inequality (21) describes the situation in which *crypto-payments are superior to money-payments*: the utility cost of purchasing good n with money exceeds the utility cost of using the crypto-currency for the same purpose. When this inequality holds, consumers will use crypto-currency to purchase the n-th good.[3] When (21) is violated, they will use money.

3.2.4 The threshold good

Under mild assumptions, there exists a unique good $n = \bar{n}$ acting as a threshold good – that is, $\bar{n}$ is interior and splits the mass of N goods in two non-empty subsets of goods purchased via different payment methods. On the demand side, the threshold condition is set by the consumers' indifference between money and crypto-currency payments: the threshold good $n = \bar{n}$ is characterized by (21) holding as a strict equality,

$$Qp^*(\bar{n}) \cdot \left(1+\varphi^C\right) = p(\bar{n}) \cdot \frac{1+\tau}{1-\delta(\bar{n})}. \tag{22}$$

3.2.5 Goods producers: no-arbitrage pricing

On the supply side, the prices $p(n)$ and $p^*(n)$ must obey a no-arbitrage condition that makes firms producing goods indifferent between receiving money or crypto-currency payments. Assume perfect competition among producers in each sector $n \in [0, N]$ and full access of firms to both means of

[3] We are arbitrarily assuming that, in case of strict equality in (21), the indifferent consumer will opt for crypto-currency payment.

payment. Price-taking firms will adopt a combination of prices, $p(n)$ and $p^*(n)$, that yields zero profits irrespective of which currency is used in the transaction. Each unit of good sold yields either $p(n)$ units of money or $p^*(n)$ units of crypto-currency that the firm needs to re-convert into money in order to remunerate the factors of production. Hence, selling the good versus crypto-currency yields a net marginal revenue of $Qp^*\left(1-\varphi^F\right)$ units of money – where the last term includes the re-conversion fee rate φ^F that the firm pays to the exchange platform for seeling crypto versus money. The *no-arbitrage condition for firms* thus reads

$$Qp^*(n)\cdot\left(1-\varphi^F\right)=p(n) \quad \text{for each } n\in[0,N]. \tag{23}$$

3.2.6 Critical condition for the threshold good

Combining the supply-side condition (23) with the demand-side condition (22), we obtain the equilibrium condition for overall no-arbitrage between money-payments and crypto-payments for goods,

$$\frac{1+\varphi^C}{1-\varphi^F}=\frac{1+\tau}{1-\delta(\bar{n})}. \tag{24}$$

Expression (24) defines a unique fixed point $\bar{n}$ under a number of circumstances. In our model we posit the following linear relationship

$$\delta(n)=\beta\cdot(n/N) \qquad \text{with } 0<\beta<1. \tag{25}$$

Assumption (25) introduces a ranking within the mass of goods: $n\in[0,N]$ becomes an index that sorts consumption goods by increasing levels of private disutility from money use. Purchasing good $n=0$ with money does not generate any direct disutility (besides the utility loss induced by pecuniary transaction costs, τ). Purchasing with money other goods bears increasing disutility as n increases. The last good in the list, $n=N$, carries the highest direct disutility from money payments, $\delta(N)=\beta$. The restriction $\beta<1$ ensures $1-\delta(N)>0$, so that the associated utility level $u_N=\ln[c(N)\cdot(1-\delta(N))]$ is well defined. From (24) and (25), we obtain the results summarized in the following

Proposition. The optimal payment method for any good $n\in[0,N]$ is determined by

$$\delta(n) \quad < \quad 1-\frac{(1+\tau)\left(1-\varphi^F\right)}{1+\varphi^C}\Longrightarrow \text{Money} \tag{26}$$

$$\delta(n) \quad \geqslant \quad 1-\frac{(1+\tau)\left(1-\varphi^F\right)}{1+\varphi^C}\Longrightarrow \text{Crypto-currency} \tag{27}$$

Given $\delta(n)=\beta\cdot(n/N)$, there are three possible scenarios. Scenario I: if $0<\beta<1-\frac{(1+\tau)\left(1-\varphi^F\right)}{1+\varphi^C}$, all the N goods are purchased using money. Scenario II: if $1-\frac{(1+\tau)\left(1-\varphi^F\right)}{1+\varphi^C}<0<\beta$, all the N goods are purchased using the crypto-currency. Scenario III: if

$$0<1-\frac{(1+\tau)\left(1-\varphi^F\right)}{1+\varphi^C}<\beta \tag{28}$$

there exists a unique threshold good $\bar{n} \in (0, N)$ such that all goods $n \in [0, \bar{n})$ are purchased using money, whereas all goods $n \in [\bar{n}, N]$ are purchased using the crypto-currency. The threshold is determined by

$$\bar{n} = \frac{1 + \varphi^C - (1+\tau)\left(1 - \varphi^F\right)}{1 + \varphi^C} \cdot \frac{N}{\beta} \tag{29}$$

Proof. Inequalities (26)-(27) follow directly by substituting (23) into (21) and solving for $\delta(n)$. Using (25) to substitute $\delta(n)$ in (24) yields result (29). Scenarios I-III follow from the parameter restrictions that would respectively imply $\bar{n} > N$, $\bar{n} < 0$ and $0 < \bar{n} < N$ in (29). ■

Scenario I arises when even good N, the one with the highest disutility from money payments, yields higher utility when purchased with money due to relatively high crypto-fees and/or relatively low transaction costs for money. Scenario II is the opposite case in which φ^C and φ^F are relatively low and/or τ is relatively high: it can only arise if pecuniary costs for money are strictly positive, $\tau > 0$, and strong enough to more than compensate for the effects of crypto-fees. Scenario III refers to equilibria with an *interior threshold good*, where both currencies (money and crypto) are used to purchase different subsets of goods. Expression (29) confirms the most intuitive properties of the threshold index, namely,

$$\frac{\partial \bar{n}}{\partial \varphi^C} > 0, \ \frac{\partial \bar{n}}{\partial \varphi^F} > 0, \ \text{and} \ \frac{\partial \bar{n}}{\partial \tau} < 0,$$

that is, the mass of goods exclusively purchased using money $\bar{n}$ is higher the higher the crypto-fee rates and the lower the consumption tax on money payments, τ.

3.3 Production of goods

Each good $n \in [0, N]$ is produced by an indefinitely large set of competitive firms – henceforth called 'sector n' – that take prices on input and output markets as given. Despite diminishing marginal returns at the firm level, learning-by-doing spillovers at the sectoral level induce constant marginal returns to capital – that is, a constant real interest rate – in the spirit of Romer (1986) and Romer (1989). Assuming identical technologies across producers of each good $n \in [0, N]$ guarantees a symmetric equilibrium where the economy's overall bundle of consumption goods is produced according to an AK technology.

Each firm exploits the production function $y(\cdot) = k(\cdot)^{\alpha} \left(\bar{a}\ell(\cdot)\right)^{1-\alpha}$ where y is output, k is physical capital, ℓ is labor, $\bar{a}$ is workers' productivity, $\alpha \in (0, 1)$ is an elasticity parameter, and $(\cdot)$ stands for firms and/or sectoral indices to simplify the notation.[4] At the firm level, labor

[4] A complete notation would require to specify the number of firms producing good n and indexing inputs at the firm and at the sectoral levels accordingly. We avoid using the complete notation by discussing exclusively the functional forms that arise from Romer's (1986) model at the sectoral level – see Romer (1986) and Romer (1989) for details.

productivity is $\bar{a}$ is taken as given and profit maximization yields the usual first-order conditions

$$r = \alpha \frac{p(n) y(n)}{k(n)}, \tag{30}$$

$$w = (1-\alpha) \frac{p(n) y(n)}{\ell(n)}, \tag{31}$$

where $y(n)$ is total output of the n-th good, $k(n)$ and $\ell(n)$ are capital and labor used in the n-th sector, r is the market rental rate of capital and w is the prevailing wage rate.[5] At the sectoral level – i.e., across all producers of the n-th good – there are learning-by-doing spillovers whereby the use of capital increases workers' productivity. We postulate the spillover function $\bar{a} = A^{\frac{1}{1-\alpha}} (k(n)/\ell(n))$ whereby the productivity of each worker increases with the capital-labor ratio in the relevant sector. The intuition is that capital use induces complementary efficiency gains: each worker uses machines and a more intense use of machines in the sector makes each unit of labor more efficient. The spillover function implies that sectoral output becomes linear in sectoral capital,

$$y(n) = Ak(n) \text{ for each } n \in [0, N], \tag{32}$$

like in standard growth models *á la* Romer (1989). Consequently, the equilibrium interest and wage rates are

$$r = p(n) \cdot \alpha A, \tag{33}$$

$$w = p(n) \cdot (1-\alpha) A \cdot k(n)/\ell(n) \tag{34}$$

Expressions (33)-(34) imply the standard functional distribution of income whereby capital rents capture a fraction α of output value while labor incomes capture the residual fraction,

$$p(n) y(n) = \underbrace{rk(n)}_{\alpha p(n) y(n)} + \underbrace{w\ell(n)}_{(1-\alpha) p(n) y(n)} \quad \text{for each } n \in [0, N]. \tag{35}$$

Importantly, the symmetric equilibrium produces price equalization and input-ratio equalization across sectors. As each firm satisfies (33) in each sector n, each good will be sold at the same price,

$$p(n) = p \text{ for each } n \in [0, N]. \tag{36}$$

Since the exchange rate Q is not good-specific, result (36) implies $p^*(n) = p^*$ for each $n \in [0, N]$ as well, which by firms' no-arbitrage pricing (23) implies

$$p^*(n) = p^*, \qquad p^* = \frac{p}{Q \cdot (1-\varphi^F)} \quad \text{for each } n \in [0, N]. \tag{37}$$

Similarly, wage equalization across firms implies the same capital-labor ratio $k(n)/\ell(n)$ in each sector in view of (34). For future reference, we define

$$L^Y \equiv \int_0^{\bar{n}} \ell(n) dn + \int_{\bar{n}}^{N} \ell(n) dn, \tag{38}$$

where L^Y is total employment in the production of consumption goods.

[5] We are assuming competitive input markets and fully mobile homogeneous inputs so that r and w are equalized across sectors producing different goods and are taken as given by each firm.

3.4 Aggregate expenditures

Since p and p^* are identical across goods – see (36) and (37) – we can write real consumption indices by consumer type and payment type as follows:

$$\begin{cases} \tilde{c}(j) = \frac{1}{p(1+\tau)} \frac{\tilde{x}}{N} & \text{and} \quad c(j) = \frac{1}{p(1+\tau)} \frac{x}{N} & \text{for each } j \in [0, \bar{n}), \\ \tilde{c}(i) = \frac{1}{p(1+\tau)} \frac{\tilde{x}}{N} & \text{and} \quad c(i) = \frac{1}{Qp^*(1+\varphi^C)} \frac{x}{N} & \text{for each } i \in [\bar{n}, N], \end{cases} \tag{39}$$

where the expressions in the top row refer to non-binary goods (purchased by crypto-less and crypto-using consumers, respectively) and those in the bottom row refers to binary goods: the only crypto-purchased quantity is $c(i)$ with $i \in [\bar{n}, N]$.

Expressions (39) imply that all crypto-less consumers purchase the same amount of each good. Crypto-using consumers, instead, purchase different quantities depending on the means of payment. Under the assumed preferences, in particular, crypto-using individuals purchase *less* units of crypto-paid goods relative to the units of goods they purchase with money. The reason is a substitution effect: crypto-payments allow the consumer to avoid the non-pecuniary costs of money-purchases and thus yield more utility for each unit of good purchased. Under the assumed preferences, the ability to extract higher utility via non-pecuniary benefits prompts agents to *reduce* the purchased quantity $c(i)$ holding the goods' expenditure share unchanged, x/N, unchanged. In fact, the relative demand (18) and the indifference condition (22) with $p^*(\bar{n}) = p^*$ and $p(\bar{n}) = p$ imply

$$\frac{c(i)}{c(j)} = \frac{p(1+\tau)}{Qp^*(1+\varphi^C)} = (1-\delta(\bar{n})) < 1 \tag{40}$$

for each $j \in [0, \bar{n})$ and each $i \in [\bar{n}, N]$. By aggregating real indices over consumption goods, we obtain individually-purchased units of non-binary and binary goods,

$$\begin{cases} \int_0^{\bar{n}} \tilde{c}(j)\, dj = \frac{\bar{n}}{p(1+\tau)} \frac{\tilde{x}}{N} & \text{and} \quad \int_0^{\bar{n}} c(j)\, dj = \frac{\bar{n}}{p(1+\tau)} \frac{x}{N} \\ \int_{\bar{n}}^{N} \tilde{c}(i)\, di = \frac{N-\bar{n}}{p(1+\tau)} \frac{\tilde{x}}{N} & \text{and} \quad \int_{\bar{n}}^{N} c(i)\, di = \frac{N-\bar{n}}{Qp^*(1+\varphi^C)} \frac{x}{N} \end{cases} . \tag{41}$$

Individual expenditures for each type of agent read

$$\tilde{x} = p(1+\tau) \int_0^{\bar{n}} \tilde{c}(j)\, dj + p(1+\tau) \int_{\bar{n}}^{N} \tilde{c}(i)\, di, \tag{42}$$

$$x = p(1+\tau) \int_0^{\bar{n}} c(j)\, dj + Qp^* \left(1+\varphi^C\right) \int_{\bar{n}}^{N} c(i)\, di. \tag{43}$$

Multiplying by the relevant population size of each consumer category, $(1-\epsilon)L$ and ϵL, we have that total spending in the economy is

$$\begin{aligned} X &= (1-\epsilon) L\tilde{x} + \epsilon Lx = \underbrace{Lp(1+\tau)\left[(1-\epsilon)\int_0^N \tilde{c}(n)\, dn + \epsilon \int_0^{\bar{n}} c(j)\, dj\right]}_{\text{gross spending on money-purchased goods} = X^m} + \\ &+ \underbrace{\epsilon LQp^*\left(1+\varphi^C\right)\int_{\bar{n}}^{N} c(i)\, di}_{\text{gross spending on crypto-purchased goods} = X^b} = X^m + X^b \end{aligned} \tag{44}$$

which distinguishes between money-paid and crypto-paid goods and specifies that these are *gross* expenditures, that is, they include consumption taxes and fees paid to the exchange platform. For future reference, we cab rewrite aggregate gross spending on crypto-paid goods as

$$X^b = \epsilon LQp^* \left(1+\varphi^C\right) \int_{\bar{n}}^{N} c\left(i\right) di = Lp\left(1+\tau\right) \frac{\epsilon}{1-\delta\left(\bar{n}\right)} \int_{\bar{n}}^{N} c\left(i\right) di \tag{45}$$

where the last term follows by substituting $Qp^* \left(1+\varphi^C\right) = p\left(1+\tau\right) / \left(1-\delta\left(\bar{n}\right)\right)$ from (22). Hence, we can alternatively rewrite (44) as

$$X = Lp\left(1+\tau\right) \left[\left(1-\epsilon\right) \int_0^N \tilde{c}\left(n\right) dn + \epsilon \int_0^{\bar{n}} c\left(j\right) dj + \frac{\epsilon}{1-\delta\left(\bar{n}\right)} \int_{\bar{n}}^{N} c\left(i\right) di\right], \tag{46}$$

which is, again, total spending in terms of money.

3.5 The exchange platform

We model the exchange platform as a competitive sector where an indefinite number of 'crypto-exchange firms' provide services to consumers and firms and bear the cost of validating these currency transactions. In this model, validation is the activity that crypto-exchange firms must perform in every exchange operation between crypto-currency and money – which includes both selling the crypto-currency to consumers and repurchasing it from final producers. The exchange platform as a whole purchases B^H units of the crypto-currency from crypto-extractors – which represents another sector employing labor: see next subsection – and employs L^H workers to perform validation activities. Since labor is homogeneous and fully mobile between the final goods' production sector and the exchange platform, the wage rate w will be equalized between these sectors.

At the aggregate level, the *monetary inflows* of the exchange platform are represented by fees charged on consumers selling money against crypto – that is, money inflows for the platform – and by fees charged on producing firms that sell crypto against money – that is, money retained from the outflows reaching final goods' producers:

$$\begin{aligned} \text{Platform money inflows} &= \epsilon L \cdot \left[Qp^* \left(1+\varphi^C\right) - Qp^* \left(1-\varphi^F\right)\right] \cdot \int_{\bar{n}}^{N} c\left(i\right) di = \\ &= \left(\varphi^C + \varphi^F\right) \cdot Q \cdot \epsilon Lp^* \cdot \int_{\bar{n}}^{N} c\left(i\right) di = \\ &= \left(\varphi^C + \varphi^F\right) \cdot Q \cdot B^H, \end{aligned} \tag{47}$$

where the last term comes from the fact that the stock of cryptocurrency in circulation must match the consumers' total demand, $B^H = \epsilon Lp^* \cdot \int_{\bar{n}}^{N} c\left(i\right) di$. The monetary outflows of the exchange platform comprise the sectoral wage bill, wL^H, and the monetary expenses QB^H associated with the purchases of crypto-currency from crypto-extractors at the wholesale exchange rate Q. This structure has two implicit assumptions. First, consumers cannot purchase the crypto-currency directly from the crypto-extractors, they need to go through crypto-exchange firms that invest the necessary amount of labor in validation activities. Second, the crypto-exchange firms' commitment to repurchase the crypto-currency in circulation from manufacturing firms versus money is honoured without any uncertainty – e.g., because such commitment is perfectly enforceable by rule of law. Zero profits in the crypto-sector thus require

$$\left(\varphi^C + \varphi^F\right) QB^H = wL^H + QB^H.$$

There are many ways to model the behavior of crypto-exchange firms consistently with the above zero-profit condition. The simplest structure hinges on linear returns to labor in validation activities. Suppose that the exchange platform comprises H competitive firms indexed by $h \in [0, H]$. Crypto-firm h purchases b_h units of crypto-currency from crypto-extractors at the wholesale rate Q, and hires $\ell_h^C + \ell_h^F$ workers to perform validation activities, where ℓ_h^C is the number of workers validating crypto-purchases by consumers and ℓ_h^F is the number of workers validating crypto-sales by manufacturing firms. The profits of the crypto-exchange firm thus read

$$\pi_h = Qb_h \cdot \left(\varphi^C + \varphi^F - 1\right) - w\ell_h^C - w\ell_h^F.$$

The validation of crypto-transactions requires an amount of work time that depends on the number of crypto-currency units to be verified. Since both types of exchange transactions involve the same number of crypto-currency units, we can set without loss of generality $\ell_h^C = \ell_h^F = \ell_h^{\cdot}$ and, accordingly, $\varphi^C = \varphi^F = \varphi^{\cdot}$. Formally, suppose that the transfer of one unit of crypto-currency in either direction requires $\xi > 0$ units of labor, so that $\ell_h^{\cdot} = \xi b_h$. We can rewrite the profits of the crypto-exchange firm as

$$\pi_h = Qb_h \cdot (2\varphi^{\cdot} - 1) - w2\ell_h^{\cdot} = Qb_h \cdot (2\varphi^{\cdot} - 1) - w \cdot 2\xi b_h. \tag{48}$$

All firms take prices Q and w as given, and compete a la Bertrand in setting the fees $\varphi^{\cdot}$ which will result in the equality between marginal revenues and marginal costs. Given linear returns, each firm will charge the equilibrium fee rate associated with the zero profit condition $Q\left(2\varphi^{\cdot} - 1\right) = w2\xi$, that is,

$$\varphi^{\cdot} = \frac{1}{2} + \xi \cdot \frac{w}{Q}. \tag{49}$$

Note that in order to satisfy the restriction $\varphi^F < 1$, parameter ξ needs to satisfy ex-post the restriction $\xi < (1/2) \cdot (Q/w)$ in equilibrium. At the aggregate level, the zero profit condition reads

$$(2\varphi^{\cdot} - 1) \cdot QB^H = wL^H. \tag{50}$$

Substituting $\varphi^{\cdot}$ from (49) into (50) yields total labor employed in the exchange platform as a function of the total crypto-currency in circulation

$$L^H = 2\xi \cdot B^H, \tag{51}$$

where B^H is determined by crypto-extractors as discussed below.

3.6 Crypto-extractors and potential supply

The model incorporates an important distinction between crypto-currency in circulation, B^H, and potential crypto-currency supply. On the one hand, the total crypto-currency in circulation is the relevant notion of supply in the market-clearing condition for goods' transactions: B^H matches the total amount of cryptocurrency units used by consumers in crypto-payments for goods,

$$\underbrace{B^H}_{\text{Crypto-currency in circulation}} = \underbrace{\epsilon L p^* \cdot \int_{\bar{n}}^{N} c(i)\, di}_{\text{Transactive demand}}. \tag{52}$$

On the other hand, the *potential supply of crypto-currency*, denoted by B^S, is a fixed number of crypto-currency units representing the stock from which the B^H units in circulation are extracted. In our model, B^S is an exogenous constant: the potential supply of crypto-currency can be thought of as a mass of virtual coins with no inherent value, costlessy created by an external entity – which bears similarities to money supply, M^S. However, differently from money supply, the potential supply of crypto-currency is not put into circulation for free and its the end-users of crypto-currency do not have direct access to B^S. Access is restricted to firms – henceforth called 'crypto-extracting firms' – that pay a fixed startup cost as well as variable "mining costs" to extract units that can be sold to crypto-exchange firms against money at the wholesale exchange rate Q. The number of crypto-extracting firms, S, is endogenously determined by free entry leading to a symmetric equilibrium with zero profits – i.e., a situation in which operative profits cover exactly the fixed startup cost.

Consider a single crypto-extracting firm indexed by $s \in [0, S]$. Setting up the firm incurs a fixed labor cost, ℓ_s^f, which can be thought of as real resources to be invested in obtaining access to extraction activities. The firm then hires ℓ_s^m workers to extract $b_s(\ell_s^m)$ units of crypto-currency from the stock B^S according to the technology

$$b_s(\ell_s^m) = \left(\bar{b} \cdot \ell_s^m\right)^{\varsigma}, \qquad 0 < \varsigma < 1, \tag{53}$$

where $\bar{b}$ is a labor efficiency parameter that firm s takes as given. The firm's profits read

$$\pi_s = Q \cdot b_s(\ell_s^m) - w\ell_s^m - w\ell_s^f = Q \cdot \left(\bar{b} \cdot \ell_s^m\right)^{\varsigma} - w\ell_s^m - w\ell_s^f \tag{54}$$

and the first order condition with respect to ℓ_s^m implies

$$\varsigma \cdot Q \cdot \left(\bar{b} \cdot \ell_s^m\right)^{\varsigma} = w\ell_s^m. \tag{55}$$

Substituting (55) back into the profit equation yields

$$\pi_s = (1 - \varsigma) \cdot Q \cdot \left(\bar{b} \cdot \ell_s^m\right)^{\varsigma} - w\ell_s^f. \tag{56}$$

The combination of price-taking behavior and decreasing marginal returns to mining, $\varsigma < 1$, generates potentially positive profits that more than compensate for the fixed cost $w\ell_s^f$. However, in this case, the entry of more firms in the extraction sector can squeeze profits by increasing the difficulty each firm faces in extraction until each firm makes zero profits. A simple way to model this outcome is to assume that the efficiency of each worker employed in extraction, $\bar{b}$, increases with the stock of potential supply B^S and decreases with the total number of workers competing for it,

$$\bar{b} \equiv \vartheta \frac{B^S}{\int_0^S \ell_s^m ds}, \qquad \vartheta > 0. \tag{57}$$

In a symmetric equilibrium where ℓ_s^m is the same for each firm, substitution of (57) into (53) yields the extraction level

$$b_s\left(\ell_s^m\right) = \left(\vartheta \frac{B^S}{S\ell_s^m} \cdot \ell_s^m\right)^{\varsigma} = \left(\vartheta \frac{B^S}{S}\right)^{\varsigma}. \tag{58}$$

Since per-firm extraction declines with the total number of firms, profits per firm decline with S. A symmetric zero-profit equilibrium will hold under free entry when the number of firm reaches the critcal level $S = \bar{S}$ given by

$$(1-\varsigma)\, Q \left(\vartheta \frac{B^S}{\bar{S}}\right)^{\varsigma} = w\ell_s^f \quad \rightarrow \quad \bar{S} = \vartheta B^S \cdot \left(\frac{1-\varsigma}{\ell_s^f} \cdot \frac{Q}{w}\right)^{\frac{1}{\varsigma}}. \tag{59}$$

Importantly, the amount of labour hired in extraction activities at the firm level is determined by the fixed cost: solving the first order condition (55) for ℓ_s^m yields

$$\ell_s^m = \varsigma \cdot \frac{Q}{w} \cdot \left(\vartheta \frac{B^S}{\bar{S}}\right)^{\varsigma} = \frac{\varsigma}{1-\varsigma} \cdot \ell_s^f. \tag{60}$$

Aggregating across firms, total employment is

$$L^S = \bar{S} \cdot \left(\ell_s^m + \ell_s^f\right) = \bar{S} \cdot \frac{\ell_s^f}{1-\varsigma} = \vartheta \left(\frac{1-\varsigma}{\ell_s^f}\right)^{\frac{1-\varsigma}{\varsigma}} B^S \cdot \left(\frac{Q}{w}\right)^{\frac{1}{\varsigma}}, \tag{61}$$

and the zero profit condition reads

$$w = \frac{B^H Q}{L^S}. \tag{62}$$

Total production can be written as

$$B^H = \bar{S} \cdot \left(\vartheta \frac{B^S}{\bar{S}}\right)^{\varsigma} = \left(\vartheta B^S\right)^{\varsigma} \left(\bar{S}\right)^{1-\varsigma} = \vartheta B^S \cdot \left(\frac{1-\varsigma}{\ell_s^f} \cdot \frac{Q}{w}\right)^{\frac{1-\varsigma}{\varsigma}}$$

which can be useful for future reference.

3.7 Aggregate income

Since there are no savings in this static economy, total expenditures must match total incomes. Distinguishing among sources of income by sector, we can rewrite aggregate incomes Y^i as

$$Y^i = w\left[L^H + L^s\right] + \left[wL^Y + rK\right] + Lg. \tag{63}$$

where the right hand side specifies the incomes received by workers employed in exchange and extracting activities, $w\left[L^{H}+L^{s}\right]$, and by owners of the inputs in goods' production, $wL^{Y}+rK$. The three components of total expenditures satisfy the following equations. First, the wage bill of the crypto-currency sector is determined by (47) together with the market-clearing and the zero profit conditions, respectively (52) and (62)

$$w\left[L^{H}+L^{s}\right]=\left(\varphi^{C}+\varphi^{F}\right)\cdot Q\cdot\epsilon Lp^{*}\cdot\int_{\bar{n}}^{N}c\left(i\right)di. \tag{64}$$

Second, total factor payments to the inputs producing consumption goods equal the market value of the resulting output sold by firms,

$$wL^{Y}+rK=p\left(1-\epsilon\right)L\int_{0}^{N}\tilde{c}\left(n\right)dn+p\epsilon L\int_{0}^{\bar{n}}c\left(j\right)dj+\left(1-\varphi^{F}\right)Q\epsilon Lp^{*}\int_{\bar{n}}^{N}c\left(i\right)di. \tag{65}$$

Third, total net transfers to households consist of lump-sum tax rebates, i.e., the government revenues from the consumption tax applied to all money-purchased goods,

$$Lg=L\tau p\left(1-\epsilon\right)\int_{0}^{N}\tilde{c}\left(n\right)dn+L\tau p\epsilon\int_{0}^{\bar{n}}c\left(j\right)dj. \tag{66}$$

It can be easily verified that Y^{i} coincides with X in expression (46), that is, the aggregate constraint requiring total expenditures to match total incomes is satisfied. By substituting (64), (65) and (66) in (63), we have

$$\begin{aligned}Y^{i} &= Qp^{*}\left(1+\varphi^{C}\right)\epsilon L\int_{\bar{n}}^{N}c\left(i\right)di+\\ &+p\left(1-\epsilon\right)L\int_{0}^{N}\tilde{c}\left(n\right)dn+p\epsilon L\int_{0}^{\bar{n}}c\left(j\right)dj+\\ &+\tau\cdot p\left(1-\epsilon\right)L\int_{0}^{N}\tilde{c}\left(n\right)dn+\tau\cdot p\epsilon L\int_{0}^{\bar{n}}c\left(j\right)dj,\end{aligned}$$

where we can substitute $Qp^{*}\left(1+\varphi^{C}\right)=p\left(1+\tau\right)/\left(1-\delta\left(\bar{n}\right)\right)$ from (22) to obtain

$$\begin{aligned}Y^{i} &= p\left(1+\tau\right)\cdot\frac{\epsilon}{1-\delta\left(\bar{n}\right)}L\int_{\bar{n}}^{N}c\left(i\right)di+\\ &+p\left(1+\tau\right)\left(1-\epsilon\right)L\int_{0}^{N}\tilde{c}\left(n\right)dn+p\left(1+\tau\right)\epsilon L\int_{0}^{\bar{n}}c\left(j\right)dj,\end{aligned}$$

that is,

$$Y^{i}=p\left(1+\tau\right)\left[\left(1-\epsilon\right)L\int_{0}^{N}\tilde{c}\left(n\right)dn+\epsilon L\int_{0}^{\bar{n}}c\left(j\right)dj+\frac{\epsilon}{1-\delta\left(\bar{n}\right)}L\int_{\bar{n}}^{N}c\left(i\right)di\right], \tag{67}$$

where the right hand side of (67) coincides with the right hand side of (46), implying $Y^{i}=X$.

Henceforth, we assume that the government rebates the revenues of the consumption tax to all consumers without distinction, i.e., in transfers of equal amounts for both crypto-less and crypto-using consumers. Equal transfers per capita, $g = \tilde{g}$, imply equal income and expenditure levels across the two categories of consumers: from (5) and (17), we obtain $x = \tilde{x}$. This implies that all individuals consume the same units of money-purchased goods regardless of the consumer type: going back to (41), we have

$$\tilde{c}(i) = \tilde{c}(j) = c(j) = c^m \text{ for each } j \in [0, \bar{n}), \tag{68}$$

where $\tilde{c}(j)$ and $\tilde{c}(i)$ are purchased by crypto-less individuals, while $c(j)$ are purchased by individuals that have access to the crypto-currency but prefer to use money for such goods. Considering $c(i)$ with $i \in [\bar{n}, N]$, instead, the units of crypto-purchased goods are strictly less than c^m in view of (40), which then implies

$$c(i) = (1 - \delta(\bar{n})) \cdot c^m = c^b \text{ for each } i \in [\bar{n}, N]. \tag{69}$$

Results (68)-(69) imply that total income/expenditure can be written in more compact form as

$$X = Y^i = Lp(1+\tau) N \cdot c^m, \tag{70}$$

which will be useful later.

4 Equilibrium

4.1 Money and Crypto-currency

Total money in circulation results from an exogenous rule whereby all factor incomes and net taxes must be paid in terms of *money* issued by the government. The rule says that the nominal money stock equals the value of total incomes, $M^s = Y^i$. As a consequence, real money supply (in terms of consumption goods) equals real income,

$$\frac{M^s}{p} = \frac{Y^i}{p} = L(1+\tau) N \cdot c^m \tag{71}$$

where the last term comes from (70). As for the crypto-currency, the market clearing condition is (52) and can be rewritten using (68)-(69) in real terms as

$$\frac{B^H}{p^*} = \epsilon L (N - \bar{n})(1 - \delta(\bar{n})) \cdot c^m \tag{72}$$

which says that the supply of crypto-currency must match the (transactive) demand for crypto-currency.

The relevant equations for the real exchange rate between money and crypto-currency are the no-arbitrage equation for producers (23) and for consumers (22), which we report here for the sake of exposition,

$$\begin{aligned}\frac{Qp^*}{p} &= \frac{1}{1-\varphi^F} &(73)\\ \frac{Qp^*}{p} &= \frac{1+\tau}{(1-\delta(\bar{n}))\cdot(1+\varphi^C)}. &(74)\end{aligned}$$

4.2 Input markets

In equilibrium, the labour market clears so as to ensure that total labor supply matches total employment,

$$L = L^Y + L^H + L^S. \tag{75}$$

Similarly, the capital market requires

$$K = \int_0^{\bar{n}} k(j)dj + \int_{\bar{n}}^N k(i)di \tag{76}$$

From the profit-maximizing conditions of goods' producers, result (36) shows that goods' prices are symmetric, $p(n) = p$ for each n, so that every firm producing goods employs the same capital-labor ratio: from (34), we have

$$k(n) = \frac{1}{(1-\alpha)A}\cdot\frac{w}{p}\cdot\ell(n) \text{ for each } n \in [0, N]. \tag{77}$$

Integrating both sides over the N goods, and substituting the aggregate labor and capital constraints (75) and (76), yields $K(1-\alpha)A = (w/p)\cdot L^Y$, which we can solve for the equilibrium real wage as

$$\frac{w}{p} = (1-\alpha)AK\cdot\frac{1}{L^Y}. \tag{78}$$

The relationship between the real wage and employment in the other sector where labor is employed, the exchange platform, follow from the zero profit condition for crypto-exchange firms in aggregate terms,

$$\frac{w}{p} = (2\varphi^{\cdot}-1)\cdot\frac{QB^H}{p}\cdot\frac{1}{L^H}. \tag{79}$$

It is useful to remember that (79) combined with the equilbrium fee (49) yields $L^H = 2\xi\cdot B^H$ independently of labor demand of crypto-extracting firms. The latter, in the present variant of the model, reads

$$\frac{w}{p} = \frac{QB^H}{p}\cdot\frac{1}{L^S}. \tag{80}$$

4.3 Goods market equilibrium

The goods' market equilibrium is characterized by the aggregate budget constraint of goods-producing firms,

$$\int_0^N py(n)\,dn = wL^Y + rK, \tag{81}$$

and by the expenditure-income equality (70). Since $y(n) = Ak(n)$ for each $n \in [0, N]$ from result (32), total production of goods equals $\int_0^N py(n)\,dn = pAK$. Using this result to substitute the left hand side of (81), and using (70) to substitute the left hand side of (81), we obtain

$$pAK = Lp \cdot [N - \epsilon (N - \bar{n}) \delta (\bar{n})] \cdot c^m,$$

from which the real consumption index c^m can be expressed as

$$c^m = \frac{AK}{L \cdot [N - \epsilon (N - \bar{n}) \delta (\bar{n})]}. \tag{82}$$

We show now have all the necessary elements to solve for the key equilbrium variables: see below.

5 Solution procedure and numerical results

5.1 Reduced system

In order to solve for the equilibrium values of the endogenous variables, we can follow a two-step procedure. First, we build a reduced system that collects a subset of the equilibrium relationships to deliver solutions for a subset of endogenous variables. Second, we use the remaining equilibrium relationships – which are implicit in the reduced system – to determine all other endogenous variables. The reduced system reads

$$\bar{n} = \frac{1 + \varphi - (1 + \tau)(1 - \varphi)}{1 + \varphi} \cdot \frac{N}{\beta}, \tag{83}$$

$$\delta(\bar{n}) = \frac{\beta}{N} \cdot \bar{n} \tag{84}$$

$$c^m = \frac{AK}{L \cdot [N - \epsilon (N - \bar{n}) \delta (\bar{n})]} \tag{85}$$

$$p = \frac{M^s}{L(1 + \tau) N \cdot c^m} \tag{86}$$

$$\frac{QB^H}{p} = \frac{\epsilon L (N - \bar{n})(1 - \delta(\bar{n}))}{1 - \varphi} \cdot c^m \tag{87}$$

$$\frac{w}{p} = (1 - \alpha) AK \cdot \frac{1}{L^Y} \tag{88}$$

$$\frac{w}{p} = (2\varphi - 1) \cdot \frac{QB^H}{p} \cdot \frac{1}{L^H} \tag{89}$$

$$\frac{w}{p} = \frac{QB^H}{p} \cdot \frac{1}{L^S} \tag{90}$$

$$L = L^Y + L^H + L^S \tag{91}$$

$$L^S = \vartheta B^S \cdot \left(\frac{1 - \varsigma}{\ell_s^f} \cdot \frac{Q}{w} \right)^{\frac{1}{\varsigma}} \cdot \frac{\ell_s^f}{1 - \varsigma} \tag{92}$$

$$\varphi = \frac{1}{2} + \xi \cdot \frac{w}{Q} \tag{93}$$

Equations (83)-(84) determine the equilibrium threshold good and the associated critical level of $\delta(n) = \delta(\bar{n})$ according to Proposition 1. Equation (85) is the final goods' market clearing condition determining consumption of money-purchased goods. Equation (86) is the cash-in-advance constraint determining the money price of final goods for a given supply of money. Equation (87) is the market clearing condition in the crypto-currency market (72) – requiring that the crypto-currency in circulation matches transactive demand from consumers – rewritten in terms of money-purchased goods. Equations (88)-(90) are the labor demand schedules of, respectively, the final goods' producers, crypto-exchange firms, and crypto-extracting firms. Equation (91) is the labor market clearing condition. Equation (92) is the zero-profit condition for crypto-extracting firms. Equation (93) is the equilibrium fee charged by crypto-exchange firms, implying zero profits in the exchange platform. This system of 11 equations determines 11 endogenous variables, namely,

$$\bar{n}, \delta(\bar{n}), c^m, p, \frac{QB^H}{p}, \frac{w}{p}, L^Y, L^H, L^S, \frac{Q}{w}, \varphi. \tag{94}$$

The equilibrium values of $(p, \frac{w}{p}, \frac{Q}{w})$ imply solutions for the levels of the key nominal variables (p, w, Q). Combining these results with the equilibrium fee φ, we obtain the real exchange rate $\frac{1}{1-\varphi} = \frac{Qp^*}{p}$ and the price of crypto-purchased goods $p^* = \frac{1}{1-\varphi}\frac{p}{Q}$. From (69), the consumption of crypto-purchased goods is $c^b = (1-\delta(\bar{n})) \cdot c^m$, and the amount crypto-currency in circulation is $B^H = \epsilon L (N-\bar{n})(1-\delta(\bar{n})) \cdot p^* \cdot c^m$. Aggregate real expenditure is $X/p = L(1+\tau) N \cdot c^m$. Finally, we can calculate welfare – and more specifically, the utility levels of crypto-using and crypto-less consumers – as

$$\begin{aligned} U &\equiv \int_0^{\bar{n}} \ln[c(j)(1-\delta(j))]\, dj + \int_{\bar{n}}^N \ln c(i)\, di = \\ &= \bar{n} \cdot \ln c^m + \int_0^{\bar{n}} \ln\left(1 - \frac{\beta}{N} \cdot n\right) dn + (N-\bar{n}) \ln c^b \end{aligned} \tag{95}$$

and

$$\begin{aligned} \tilde{U} &\equiv \int_0^N \ln[\tilde{c}(n) \cdot (1-\delta(n))]\, dn \\ &= N \cdot \ln c^m + \int_0^N \ln\left(1 - \frac{\beta}{N} \cdot n\right) dn, \end{aligned} \tag{96}$$

respectively. A numerical illustration is reported below.

5.2 Benchmark equilibrium: a numerical illustration

We use the reduced system (83)-(93) to numerically evaluate the equilibrium levels of the endogenous variables. The calibration of the exogenous constants in Table 1 adheres to the mathematical constraints underlying the construction of our one-period model. The baseline results are then reported in Table 1 and the subsequent ones. Figure 1 illustrates the variation in utility levels over the goods space for crypto-using and crypto-less consumers, as formulated in expressions 95 and 96. Overall, our baseline results align with the expected signs and magnitudes of the model's key variables. For instance, the threshold good $\bar{n}$ satisfies the restriction implied by the proposition in subsection 3.2.6. The distribution of the stock of goods purchased with either means of payment is consistent with real-world observations, where crypto-currencies are marginally used in goods transactions. In terms of utility gains, crypto-using consumers strictly dominate crypto-less consumers. The latter finding gives us a strong basis to study how various policy and non-policy decisions affect the welfare level of the two consumer types in our economy. Below, we assess the resulting effect of a partial change in some of the key exogenous constants around the benchmark equilibrium solutions.

Table 1: Exogenous constants and their values

Variable/Parameter	τ	N	β	M	L	ϵ	A	K	α	ς	l_s^f	B^S	ξ	ϑ
Value	0.05	50	0.75	1000	200	0.5	0.10	10000	0.4	0.25	10	1000	0.02	1

5.3 Policy shocks: money growth, crypto-currency supply and tax hike

Effects of money growth. Given the chosen exogenous constants, a 10% increase in money growth pushes up the market price of money-purchased goods and the nominal wage by a comparable proportion. This results in an appreciation of the nominal exchange rate between money and crypto-currency to maintain identical expenditure levels between crypto-less and crypto-using consumers. Real expenditures, demand for consumption goods, and overall consumer welfare remain unchanged under the studied monetary shock. The model's prediction aligns with the real business cycle literature, where an aggregate money supply shock affects only prices in the economy, in this case p and w.

Effects of crypto-currency growth. In equilibrium, a positive crypto-currency supply shock affects both the nominal and real variables of the model. Initially, the increase in crypto-currency circulation drives up the demand for exchange rate conversion service, leading to a rise in φ. The shock also triggers a rise in the market price of crypto-purchased goods. In response, consumers substitute crypto-purchased goods with money-purchased goods. This price inflation reduces real aggregate consumption in the economy and decreases real expenditures. In the labour sector, employment rises in the goods production and the crypto-exchange sectors whereas the extraction sector sees a halt in employment activity. Given that labour compensation is homogeneous across sectors, the lower employment activity in the extraction sector forces the labour market to clear at a lower nominal wage compared to the benchmark simulation. The crypto shock deteriorates welfare for both crypto-using and crypto-less consumers. The welfare loss for money-using consumers derives from consuming goods with an increased disutillity penalty. As explained above, the crypto price inflation enlarges the subset of goods purchased with money. The downside comes with the fact that each additional good added to the subset of money-purchased good bears higher disutility for the consumer and reduces the associated utility. In the case of crypto-using consumers, the welfare loss reflects the impact of inflation on the real consumption. Overall, the crypto-currency supply shock has real effects and impacts consumer welfare, as reflected in the changes in the utility levels in Table 2.

Effects of a consumption tax hike. A 10% consumption tax negatively impacts the quantity of money-purchased goods, putting downward pressure on the market price of supplied goods. The resulting positive income effect boosts the real consumption in the economy. However, the demand for crypto-purchased goods grow faster than money-purchased ones under the 10% tax increase. On the labour side, we observe a contraction in employment in goods production, leading to a decline in the nominal wage rate. Meanwhile, the increased demand for crypto-purchased goods combined with a higher demand for crypto-currencies raise employment in exchange and extraction activities. In aggregate terms, real expenditures rise due to the boost in real consumption. It is worth noting that the specified shock pushes down crypto fees. This explains a scale effect in the crypto sector where higher activities attract new entrants and drives down the market price for service. In conclusion, both consumer types enjoy higher utility as a result of stronger purchasing power as reported in Table 2.

Table 2: Benchmark results and policy shock analysis

Variable	**Baseline**	ΔM^S	ΔB^S	$\Delta \tau$
φ	0.5084	0.5084	0.5086	0.5083
$\bar{n}$	43.8506	43.8506	43.8633	43.7391
$\delta(\bar{n})$	0.6578	0.6578	0.6579	0.6561
c^m	0.1042	0.1042	0.1042	0.1043
c^b	0.0357	0.0357	0.0356	0.0359
p	0.9139	1.0052	0.9139	0.9089
p^*	0.2635	0.2635	0.2701	0.2616
Q	7.0554	7.7610	6.8852	7.0673
w	2.9488	3.2437	2.9487	2.9378
L^Y	185.9434	185.9434	185.9678	185.6357
L^H	0.2311	0.2311	0.2363	0.2349
L^S	13.8255	13.8255	13.7959	14.1294
B^H	5.7784	5.7784	5.9082	5.8734
$\bar{S}$	1.0369	1.0369	1.0347	1.0597
$\frac{w}{Q}$	0.4180	0.4180	0.4283	0.4157
$\frac{Qp^*}{p}$	2.0340	2.0340	2.0349	2.0338
$\frac{w}{p}$	3.2268	3.2268	3.2264	3.2321
$\frac{QB^H}{p}$	44.6120	44.6120	44.5105	45.6680
$\frac{B^H}{B^S}$	0.0058	0.0058	0.0054	0.0059
$\frac{X}{P}$	1094.2611	1094.2611	1094.1793	1100.1924
U	-139.0447	-139.0447	-139.0519	-138.9817
$\tilde{U}$	-139.9599	-139.9599	-139.9636	-139.9271

5.4 Population shocks versus crypto-currency access

Effects of an increase in L. An increase in population size exerts pressure on the labour market in the form of excess supply as reported in Table 3. The real wage shifts downward to reflect the combined effect of a lower nominal wage on the labour market and inflationary pressures stemming from crypto-purchased goods. The population shock increases the demand for the mass of goods purchased with crypto-currencies and lowers the crypto fees, through the same scale effect mechanism explained in the section above. Following the shock, real expenditures go up, reflecting the volume effect generated by a higher population size. On the crypto-currency side, the population growth leads to higher demand of currency for transactions. In a nutshell, a population shocks affects all segments of crypto-backed activities. The shock is welfare-deteriorating for both crypto-using and crypto-less consumers, which comes from the contraction in real consumption induced by the reduction in real purchasing power.

Effects of an increase in ϵ. A higher penetration of crypto-currencies as a means of payment increases the consumption of crypto-purchased goods. The market reacts by putting downward pressures on the price of money-purchased goods. A direct consequence of a positive shock to ϵ is a rise in employment in crypto-related activities, as shown in Table 3. In other words, the shock causes a shift in labour force from goods production to crypto-related occupations. As in the above analysis, higher activities in crypto-backed activities lower transaction fees charged by exchange platforms. The deflationary impact on both types of goods increases the purchasing power of consumers in the economy. A higher real wage drives up expenditures and positively affects the utility derived by different types of consumers.

5.5 Technology shocks

Effects of an increase in A. The 10% productivity shock does not affect the distribution of goods across the different types of consumers as reported in table 4. The effects are similar to those observed in related neoclassical models, where technology-driven shocks improve production processes and increase aggregate output in the economy. The higher supply of goods exerts downward pressure on prices and strengthens real wages. This, in turn, increases real expenditures by a comparable percentage, leading to a positive impact on the utility enjoyed by different types of consumers.

Effects of an increase in ℓ_s^f. Higher fixed capital investment in the extraction activity reduces transaction fees on crypto-purchased goods. This can be interpreted as a network effect, where exchange platforms respond by improving their efficiency in validating crypto-backed transactions, thereby lowering costs for other economic agents in the economy. The increased investment triggers a reallocation of the labour force, leading to higher employment in the extraction industry. Nominal wages readjust to clear the labour market at a higher rate. As a result of the higher wage bill, some firms exit the extraction industry, negatively impacting the stock of crypto-currency in circulation. The combined effect of higher nominal wages and lower prices boosts aggregate consumption and

Table 3: Benchmark results and population shock analysis

Variable	**Baseline**	ΔL	$\Delta \epsilon$
φ	0.5084	0.5082	0.5082
$\bar{n}$	43.8506	43.8382	43.8386
$\delta(\bar{n})$	0.6578	0.6576	0.6576
c^m	0.1042	0.0947	0.1047
c^b	0.0357	0.0324	0.0358
p	0.9139	0.9138	0.9099
p^*	0.2635	0.2827	0.2553
Q	7.0554	6.5725	7.2465
w	2.9488	2.6809	2.9582
L^Y	185.9434	204.5115	184.5604
L^H	0.2311	0.2487	0.2481
L^S	13.8255	15.2398	15.1915
B^H	5.7784	6.2163	6.2015
$\bar{S}$	1.0369	1.1430	1.1394
$\frac{w}{q}$	0.4180	0.4079	0.4082
$\frac{Qp^*}{p}$	2.0340	2.0332	2.0332
$\frac{w}{p}$	3.2268	2.9338	3.2510
$\frac{QB^H}{p}$	44.6120	44.7109	49.3871
$\frac{B^H}{B^S}$	0.0058	0.0062	0.0062
$\frac{X}{p}$	1094.2611	1094.3409	1098.9790
U	-139.0447	-143.8032	-138.8264
$\tilde{U}$	-139.9599	-144.7218	-139.7448

expenditures. Overall, the investment shock improves the welfare of both crypto-using and crypto-less consumers.

6 Conclusion

In conclusion, our model sheds light on the dynamics of currency competition between fiat money and crypto-currencies, highlighting the conditions under which each form of currency is used for transactions. We demonstrate that the coexistence of both currencies hinges on the relative transaction costs, tax policies, and privacy benefits associated with each. The introduction of a threshold good, where consumers are indifferent between fiat and crypto-currency, reveals the intricate balance between the two currencies in the market. While fiat money remains neutral in our framework, crypto-currencies introduce real economic effects due to the costs associated with mining and its impact on labor distribution. Our findings suggest that the adoption of crypto-currencies has broader macroeconomic implications, particularly through changes in consumption patterns, labor reallocation, and sectoral employment. Future research should explore how these dynamics evolve over time and under different monetary policy regimes, considering the growing role of digital currencies in the global economy.

Table 4: Benchmark results and technology shock analysis

Variable	Baseline	ΔA	Δl_s^f
φ	0.5084	0.5084	0.5078
$\bar{n}$	43.8506	43.8506	43.8143
$\delta(\bar{n})$	0.6578	0.6578	0.6572
c^m	0.1042	0.1146	0.1042
c^b	0.0357	0.0392	0.0357
p	0.9139	0.8308	0.9137
p^*	0.2635	0.2395	0.2445
Q	7.0554	7.0554	7.5910
w	2.9488	2.9488	2.9493
L^Y	185.9434	185.9434	185.8737
L^H	0.2311	0.2311	0.2162
L^S	13.8255	13.8255	13.9101
B^H	5.7784	5.7784	5.4044
$\bar{S}$	1.0369	1.0369	0.9484
$\frac{w}{q}$	0.4180	0.4180	0.3885
$\frac{Qp^*}{p}$	2.0340	2.0340	2.0316
$\frac{w}{p}$	3.2268	3.5495	3.2280
$\frac{QB^H}{p}$	44.6120	49.0731	44.9019
$\frac{B^H}{B^S}$	0.0058	0.0058	0.0054
$\frac{X}{p}$	1,094.2611	1,203.6872	1,094.4946
U	-139.0447	-134.2792	-139.0243
$\tilde{U}$	-139.9599	-135.1944	-139.9492

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